The Hafele-Keating experiment and internal detection of uniform translational motion

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Abstract

Several experiments are proposed to exploit the asymmetrical time dilation effect observed in the Hafele-Keating experiment. In the first, it is shown how the angular velocity vector, $\vec{\Omega}$, of the Earth may be determined by observation of time intervals recorded by airborne clocks following Great Circle routes with different orientations and directions. Similar time intervals recorded on outward and return flights over short straight routes enable the determination of the velocity of an arbitary point on the Earth's surface, or the relative velocity of two inertial frames in free space. Finally, time intervals recorded by a clock moving relative to a ship enable both the speed of the ship relative to the Earth and the local velocity of the surface of the Earth relative the Earth-centered (non-rotating) inertial frame to be determined. These experiments demonstrate that, contrary to some statements of the Special Relativity Principle, internal detection of uniform rectilinear motion is possible.

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1 Introduction

The Hafele-Keating experiment (HKE) performed in October 1971 [1, 2] compared time intervals recorded by four caesium-beam atomic clocks, carried around the world in commercial aircraft, to those recorded by precision reference clocks at the U.S. Naval Laboratory. In the analysis of the experiment, the detailed flight paths of the aircraft on the West–to–East (W–E) and East–to–West (E–W) flights were taken into account in the prediction of special relativistic (SR) and general relativistic (GR), or gravitational, effects. Denoting the time intervals recorded by the Earthbound (airborne) clocks by T' (T'') the following predictions were obtained [1]

$$\Delta T'(\mathrm{W-E}) \equiv T''(\mathrm{W-E}) - T'(\mathrm{W-E}) = 144 \pm 14\mathrm{ns}, \qquad \mathrm{(GR)}$$

$$= -184 \pm 18\mathrm{ns}. \qquad \mathrm{(SR)}$$

$$\Delta T'(\mathrm{E-W}) \equiv T''(\mathrm{E-W}) - T'(\mathrm{E-W}) = 179 \pm 18\mathrm{ns}, \qquad \mathrm{(GR)}$$

$$= 96 \pm 10\mathrm{ns}. \qquad \mathrm{(SR)}$$

The physical origin of the GR effect is gravitational blue-shift of the airborne clock relative to one on the surface of the Earth due to its higher gravitational potential [1] and so is determined by the mean altitude of the aircraft. The SR contribution to $\Delta T'$ arises from the time dilation (TD) effect. In calculating it, the concept of 'coordinate time' [3, 4], or the equivalent one of 'base' and 'travelling' frames [5, 6] to be explained below, is essential. Coordinate time is registered by a hypothetical clock in a non-rotating frame[2], comoving with the centre of the Earth, sufficiently distant that any effect of the Earth's gravitational field may be neglected. This is the Earth Centered Inertial (ECI) frame which is also used in the analysis of SR effects in the GPS system [7]. The experimental results of the HKE were found to be in good agreement with the above predictions [2]:

$$\Delta T'(\mathrm{W-E}) = -59 \pm 10\mathrm{ns}, \qquad \mathrm{(Experiment)}$$

$$= -40 \pm 23\mathrm{ns}. \qquad \mathrm{((GR)+(SR))}$$

$$\Delta T'(\mathrm{E-W}) = 273 \pm 7\mathrm{ns}, \qquad \mathrm{(Experiment)}$$

$$= 275 \pm 21\mathrm{ns}. \qquad \mathrm{((GR)+(SR))}$$

Other experiments were performed in which SR and GR effects near the Earth were measured and demonstrated to be in good agreement with theoretical predictions [8, 9, 10, 11]. In the experiment of Vessot *et al* 1980 [9], the SR and GR corrections to the rate of a hydrogen-maser clock flown in a rocket on a ballistic trajectory with a maximum altitude of 10^4km were of comparable magnitude and opposite sign so that exact cancellation of the effects was observed during both the ascent and descent of the rocket. In the work described by Alley [10] the altitude-dependence of the GR effect was tested by flying an aircraft carrying an array of caesium-beam atomic clocks on a closed path with a racetrack-like configuration over Chesapeake Bay in the State of Maryland USA. The

[1] The gravitational potential due to the Earth is negative and, assuming spherical symmetry, on and above the surface of the Earth is proportional to $1/r$ where r is the distance from the center of the Earth.

[2] In the approximation that the rotation of the Earth around the centroid of the Solar System is neglected, this frame is an inertial one.

same array of clocks was also flown from Washington DC to Thule, Greenland, and back, to check the predicted almost-exact cancellation [12, 13] of SR and GR effects on the rate of clocks placed on the Earth's geoid at different latitudes. In the Spacelab experiment NAVEX [11] a caesium-beam atomic clock carried by a space shuttle in low-Earth orbit was compared to similar ground-based clocks, and the sum of the SR and GR predictions for the relative rate of the clocks was verified. The TD effect and the general relativistic blue-shift give very large effects on the rates of the satellite-borne clocks of the GPS system [7] which are compensated for by adjusting the rates of the clocks on the surface of the Earth before they are sent into orbit. The HKE is however unique in its first demonstration of asymmetric time dilation effects for clocks on W$-$E and E$-$W paths. This is because [4] satellite-borne clocks have speeds much greater than Earth-bound ones, so that the time dilation asymmetry in Eq. (2.8) below is imperceptible for them. Only in the HKE are the speeds, in the ECI frame, of the moving and Earth-bound clocks of similar magnitude. The demonstration, in the HKE, that *the relative rate of two clocks does not depend only on their relative velocity* is of a crucial importance for a correct understanding of temporal predictions in SR [3, 4, 5, 6].

Asymmetrical time dilation is pure SR effect that can be seen [14] to be an immediate consequence of the Minkowski metric equation of SR for space-time intervals:

$$(\Delta s)^2 = c^2(\Delta\tau)^2 = c^2(\Delta t)^2 - (\Delta\vec{x})^2. \tag{1.1}$$

Suppose that two clocks C', C" with proper frames S', S" registering times t', t'' have the world lines $\Delta\vec{x}(\mathrm{C}') = \vec{v}(\mathrm{C}')\Delta t$, $\Delta\vec{x}(\mathrm{C}'') = \vec{v}(\mathrm{C}'')\Delta t$, in the frame S. It then follows from (1.1) that:

$$c^2(\Delta t')^2 = (c^2 - v(\mathrm{C}')^2)(\Delta t)^2, \quad c^2(\Delta t'')^2 = (c^2 - v(\mathrm{C}'')^2)(\Delta t)^2 \tag{1.2}$$

where $v \equiv |\vec{v}|$, so that

$$\Delta t = \gamma(\mathrm{C}')\Delta t' = \gamma(\mathrm{C}'')\Delta t'' \tag{1.3}$$

where

$$\gamma(\mathrm{C}') \equiv \frac{1}{\sqrt{1-\frac{v(\mathrm{C}')^2}{c^2}}}, \quad \gamma(\mathrm{C}'') \equiv \frac{1}{\sqrt{1-\frac{v(\mathrm{C}'')^2}{c^2}}}$$

It is immediately obvious, on inspection of the last member of (1.3), that the ratio of the proper time intervals $\Delta t'$, $\Delta t''$, registered by the clocks C' and C", depends not on the relative velocity $|\vec{v}(\mathrm{C}'') - \vec{v}(\mathrm{C}')|$ of the clocks but on the separate and independent values of $v(\mathrm{C}')$ and $v(\mathrm{C}'')$. These velocities of the clocks, are specified in the frame S, the 'base frame' in which the initial conditions of the problem are defined [5, 6].

Before the HKE was performed it was asserted by Schlegel [15] that the asymmetric time dilation effect of Eq.(1.3) was in contradiction with special relativity and that time dilation can depend only on the *relative velocity* of the two clocks which are observed. In reply, Hafele defended [16] the use of coordinate time in the ECI frame in the calculations. The results of the HKE [2] showed clearly that Hafele was right and Schlegel was wrong. A subsequent paper by Schlegel [17] attempted a reanalysis of the HKE by introducing considerations of the Sagnac effect for photons —a physically distinct experiment (see Section 2 below)— and clock synchronisation procedures. Since the theoretical predictions for the HKE are of *time intervals* recorded by separate cocks, the question of their relative synchronisation, cannot, even in principle, have any effect on the predictions.

More recently it was shown [18] that, if a different frame than the ECI (for example the barycentric frame of the Solar System) is used to specify 'coordinate time', predictions at variance with those of Hafele are obtained. The results of the HKE and the correct operation of the GPS where the ECI frame is also used to calculate relativistic effects [7] seems then to indicate that the ECI is a privileged frame for the specification of coordinate time. This is a problem which merits further consideration, but lies beyond the scope of the present paper which employs only Hafele's theory that is verified both by the HKE and the correct functioning of the GPS.

In the present paper, SR effects in a simplified version of the HKE will be considered in which the aircraft moves with constant speed along Great Circles, the normal to which makes a variable angle ψ with the axis of rotation of the Earth. GR effects are neglected. Since, however, the important quantities in the analyses considered are double differences such as[3]:

$$\Delta T''(+-) \equiv \Delta T'(\mathrm{W}-\mathrm{E}) - \Delta T'(\mathrm{E}-\mathrm{W}). \tag{1.4}$$

GR contributions will cancel provided the average altitude of the aircraft is constant during its flights. There remains a GR *correction* to the value of $\Delta T''(+-)$ which is tiny in comparison with the experimental uncertainties of the time interval measurements performed in the HKE. This correction is evaluated in the full GR analysis of Appendix B, and is given in Section 2 below. It applies also to the experiments described in Sections 3 and 5, but not to the free-space experiment of Section 4 where the SR formula (2.7) below gives an exact (to all orders in β) prediction.

In the HKE the frames S' and S" are not inertial frames —they are subjected to uniform transverse accelerations. It was demonstrated, by observation of time-dilated decay lifetimes of ultra-relativistic muons with $\gamma = 29$ ($\gamma \equiv 1/\sqrt{1-(\vec{v}/c)^2}$) in near-circular orbits in a storage ring at CERN [19] that the TD effect is the same as for muons in an inertial frame with velocity equal to the magnitude, $|\vec{v}|$, of their average velocity $\vec{v}$, in the presence of a transverse acceleration of $10^{19}g$. It is therefore assumed, in the following, that S' and S" may be considered to be inertial frames for calculations of the TD effect. A similar assumption is made in the calculation of SR corrections due to TD effects for the satellite-borne clocks of the GPS system [7].

The plan of this paper is as follows: In the following section it is shown how observations of $\Delta T''(+-)$ for Great Circle flights, as a function of ψ, enable the angular momentum vector, $\vec{\Omega}$, of the Earth to be determined from purely internal measurements of differences of time intervals recorded by airborne and Earthbound clocks. In Section 3 a method is described to determine the speed and direction of motion of an arbitrary point on the Earth's surface by local measurements, using a variable which is a generalistation of $\Delta T''(+-)$, also by measuring time intervals recorded by airborne clocks and Earthbound clocks. Section 4 contains a straightforward generalisation to three spatial dimensions of

[3]Note that, since T' cancels on the right side of (1.4), in the simplified version of the HKE considered here where $T'(\mathrm{W}-\mathrm{E}) = T'(\mathrm{E}-\mathrm{W})$, it is actually equal to $T''(\mathrm{W}-\mathrm{E}) - T''(\mathrm{E}-\mathrm{W})$. If the W-E and E-W flights start and end simultaneously, $\Delta T''(+-)$ is given directly by the difference, before and after the flights, of the difference of the readings of the airborne clocks. No comparison with an Earth-bound clock is then required.

the analysis of the previous section, in order to measure internally the relative vectorial velocity of of two inertial frames in free space. Section 5 shows how both the velocity of a ship relative to the surface of the Earth (as considered by Galileo) as well as the local velocity of the surface of the Earth may, in principle, be derived by comparing time intervals recorded by clocks at rest on, and moving relative to, the ship —again, therefore by purely internal measurements. It is important to note that the velocities that are 'internally measured' in all of the above experiments are, in no sense, 'absolute' ones but relative velocities between reference frames in well-defined space-time experiments. The final section contains a summary and conclusions.

2 Internal measurement of the angular velocity $\vec{\Omega}$ of the Earth using an aircraft and two clocks

In the following discussion, only SR effects are considered, and the Earth is assumed to be exactly spherical and of radius R. For the case of equatorial circumnavigation, the initial conditions of the experiment are completely specified by two parameters, $v_{\rm E} \equiv v({\rm C}') \equiv \Omega R$, which is the speed of the clock, C$'$ (with proper frame S'), at rest on the surface of the Earth, in a non-rotating inertial frame, S, comoving with the centre of the Earth, and $v'_{\rm A} \equiv v'({\rm C}'')$, which is the speed of the airborne clock, C$''$, (with proper frame S") in the frame S'. For the analysis of the experiment it is necessary to introduce the coordinate time recorded by a hypothetical clock C at rest in S. In the nomenclature for space-time experiments recently introduced by the present author [5, 6] the frame S is termed the 'base frame' of the experiment while S' and S" constitute different 'travelling frames'. The base frame is defined as the one in which the clock is at rest in a time dilation (TD) experiment. If, in such an experiment, a moving clock is seen to run slow (fast) then the corresponding observer is in the base (travelling) frame of the experiment. In the experiments to be discussed below, the time [4], t, registered by the clock C, remains hypothetical since no observations of events in the external inertial frame S are performed. It is shown, however, that purely internal observations of the times t' and t'' recorded by C$'$ and C$''$ respectively are sufficient to establish the travelling nature of the latter frames and to determine the base frame velocity $v_{\rm E}$ and hence Ω. It will be seen that, in the lowest order approximation, knowledge of $v'_{\rm A}$ is needed only to determine the value of R.

The experiments to be considered in this section are generalisations of the HKE in which the aircraft moves with constant speed along an arbitrary Great Circle and the Earthbound clock C' is situated at an arbitrary (and initially unknown) position on the Earth's surface. A solution is sought to the following problem, supposing that the surface of the Earth may be considered to be perfectly spherical:

`Determine the size, axis of rotation, and speed of rotation of the Earth from purely internal observations of two clocks, one at rest on the surface of the Earth and the other in an aircraft that moves at constant speed` $v'_{\rm A}$

[4] The word 'time', used without qualification, means a number, registered by a clock at some instant, also commonly called the 'epoch' of the clock.

relative to the surface of the Earth, along a Great Circle.

It is also assumed that the clock C$'$ may be moved in an arbitrary manner (transported, for example, by the aircraft) over the surface of the Earth. Circumstances under which the solution of the problem would be of practical importance would be, for example, if the whole Earth were blanketed in an opaque cloud following an asteroid collision, or a massive volcanic eruption, so that the Sun and fixed stars were no longer visible from the Earth, and it was required to find the the axis of rotation of the Earth, i.e. the positions of the North and South Poles.

The geometry, in the case that the angle between the Earth's axis of rotation (z'-axis) and the normal to the plane of the Great Circle followed by the aircraft (z-axis) is ψ, is shown in perspective view in Fig. 1. The common x,x' axis is perpendicular to the plane spanned by the z and z' axes, while the y and y' axes complete right-handed Cartesian coordinate systems. Projections into the x,y and y,z planes are shown in Fig. 2a and 2b respectively. The position of the aircraft, along the Great Circle, is specified by the azimuthal angle ϕ. The angle, θ, between the direction of flight of the aircraft and the tangent vector, in the W$-$E direction, to a circle of fixed latitude, is given by the relation, derived in Appendix A

$$\cos\theta = \frac{\cos\psi}{[1-\sin^2\psi\sin^2\phi]^{\frac{1}{2}}} \tag{2.1}$$

while the geometry of Fig. 2b gives the latitude angle, λ_0, at P, in terms of ψ and ϕ_0 as

$$\sin\lambda_0 = \sin\psi\sin\phi_0. \tag{2.2}$$

The differential TD relations for the clocks C$'$ and C$''$ at an arbitrary point on the great circle with latitude angle λ are (c.f. Eq. (1.3)):

$$dt = \gamma(\mathrm{C}',\lambda)dt' = \gamma(\mathrm{C}'',\lambda,\theta)dt'' \tag{2.3}$$

where

$$\gamma(\mathrm{C}',\lambda) \equiv \frac{1}{\sqrt{1-\beta(\mathrm{C}',\lambda)^2}}, \quad \beta(\mathrm{C}',\lambda) \equiv \frac{v_{\mathrm{E}}(\lambda)}{c} \equiv \frac{\Omega R\cos\lambda}{c}, \quad v_{\mathrm{E}} \equiv v_{\mathrm{E}}(\lambda=0) \tag{2.4}$$

and c is the speed of light in free space. Since the TD factor, γ, transforms as the temporal component of the dimensionless velocity four-vector: $(\gamma,\gamma\vec{\beta})$, it follows that:

$$\gamma(\mathrm{C}'',\lambda,\theta) = \gamma(\mathrm{C}',\lambda)[\gamma'(\mathrm{C}'') + \beta(\mathrm{C}',\lambda)\beta'(\mathrm{C}'')\gamma'(\mathrm{C}'')\cos\theta] \tag{2.5}$$

where

$$\gamma'(\mathrm{C}'') \equiv \frac{1}{\sqrt{1-\beta'(\mathrm{C}'')^2}}, \quad \beta'(\mathrm{C}'') \equiv \frac{v'_{\mathrm{A}}}{c}. \tag{2.6}$$

Combining (2.3) and (2.5) gives:

$$dt'' = \frac{\gamma(\mathrm{C}',\lambda)}{\gamma(\mathrm{C}'',\lambda,\theta)}dt' = \frac{dt'}{\gamma'(\mathrm{C}'')[1+\beta(\mathrm{C}',\lambda)\beta'(\mathrm{C}'')\cos\theta]}. \tag{2.7}$$

Retaining only O(β^2) terms on the right side of (2.7) and making use of (2.1) and the generalisation of (2.2), for an arbitrary point on the Great Circle, gives:

$$dt'' = \left[1 - \frac{\beta'(\mathrm{C}'')^2}{2} \mp \beta(\mathrm{C}')\beta'(\mathrm{C}'')\cos\psi\right]dt' \tag{2.8}$$

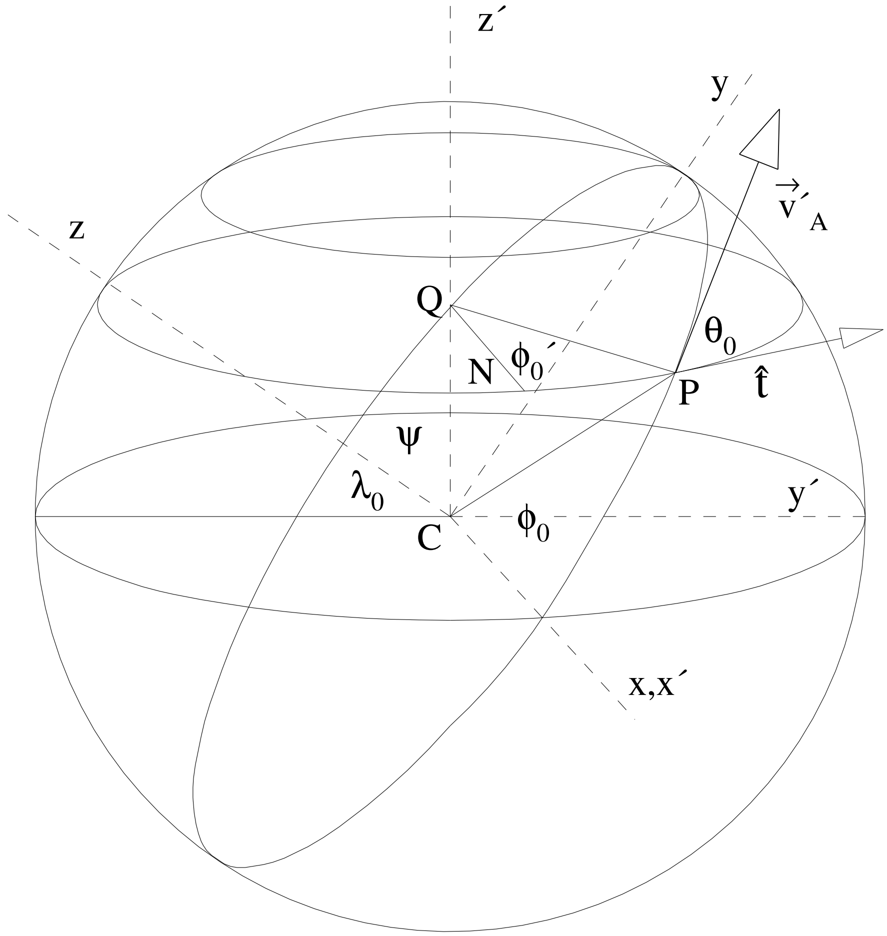

Figure 1: *Perspective view of the Great Circle path followed by the aircraft with velocity $\vec{v}'_A$ relative to the fixed point P on the surface of the Earth. The normal to the Great Circle (the $z-$axis) is inclined at an angle ψ relative to the polar $z'-$axis. The unit vector $\hat{t}$ is the tangent, in the W$-$E direction, at P, to the circle of fixed latitude λ_0. The $x, x'-$axis is normal to the plane spanned by the $z-$ and $z'-$axes and its projection into the circle of fixed latitude is QN. The angle between $\vec{v}'_A$ and $\hat{t}$ is θ_0, and ϕ_0, ϕ'_0 are azimuthal coordinates of P along the Great Circle and the circle of fixed latitude, respectively. The x, x' and y' axes lie in the Earth's equatorial plane, the x, x' and y axes in the plane of the Great Circle.*

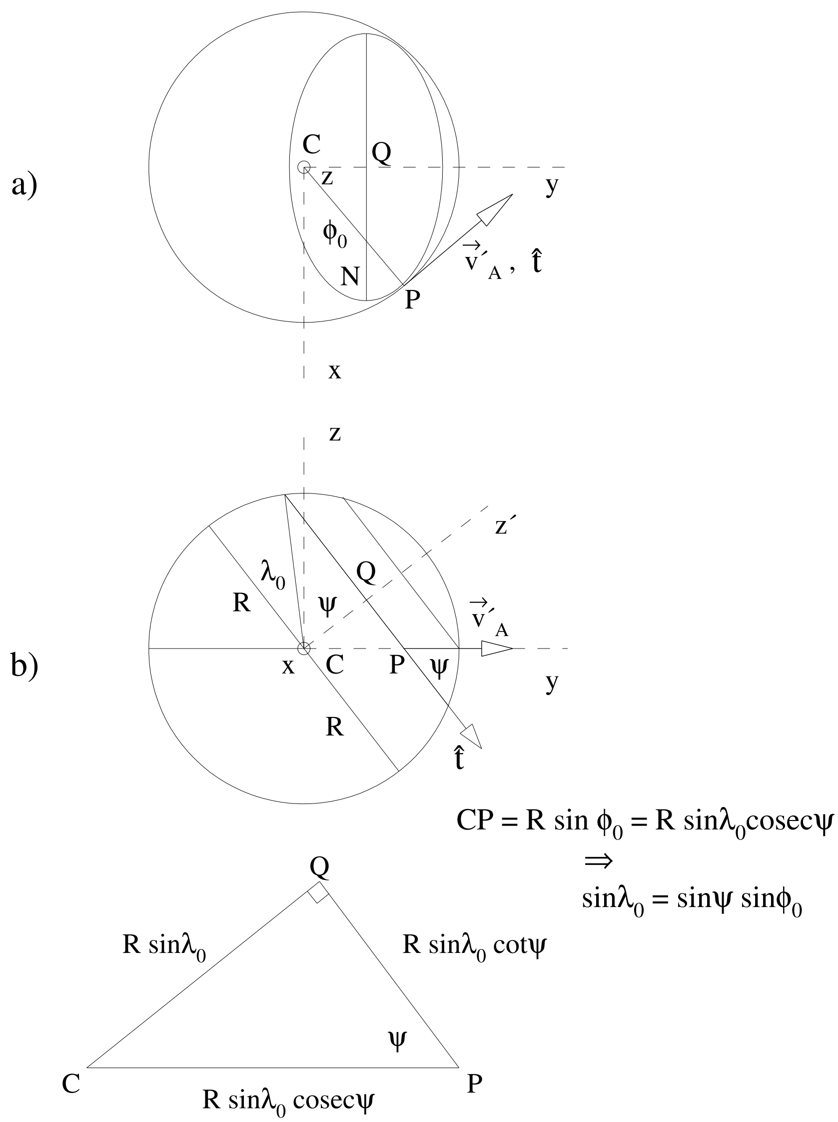

Figure 2: *a) x, y and b) y, z projections of the configuration of Fig.1. R is the radius of the Great Circle in a) and of the Earth in b).*

where $\beta(\mathrm{C}') \equiv \beta(\mathrm{C}', \lambda = 0)$ and the $-$ $(+)$ signs correspond to $cos\theta > (<)0$. In (2.8) the relation (see Appendix A) $\beta(\mathrm{C}', \lambda)\cos\theta = \beta(\mathrm{C}')\cos\psi$ has been used. Since the aircraft moves with uniform speed around the Great Circle, $dt' = (R/v'_{\mathrm{A}})d\phi$. Also (2.1) and (2.2) may be used to derive the following relation (see Appendix A) between ψ and the values, θ_0 and λ_0, of θ and λ, at the point P:

$$\cos\psi = \cos\theta_0 \cos\lambda_0. \tag{2.9}$$

The formula (2.8) which is the basis for all the calculations presented in the present and subsequent sections of the present paper was first derived for the special case: $\cos\theta > 0$ and $\psi = 0$ by Hafele [3, 4]. Since it gives the ratio of infinitesimal proper time intervals, it does not depend on the shape of the world lines followed, in the frame S, by the clocks C' and C". In particular it is equally valid for the circular world lines considered in the present section and the straight ones considered in Sections 3, 4 and 5 below, where the proper frames of C' and C" are inertial ones. Hafele's GR calculation, yielding (2.8), in the limit of a vanishing or constant gravitational field, is recalled in Appendix B below. It was also derived as a pure SR calculation from the Minkowski metric (as in the derivation of Eq. (1.3) in the Introduction of the present paper or by Hafele (Appendix of [4]).

Labelling the Great Circle followed by the aircraft as '+' if $\cos\theta_0 > 0$ (i.e. one with W–E motion) and '$-$' if $\cos\theta_0 < 0$ (i.e. one with E–W motion) the round trip time T'' as recorded by C$''$ is given by (2.8) as:

$$\begin{aligned} T''(\theta_0, \lambda_0, \pm) &= \frac{R}{v'_{\mathrm{A}}}\int_{\phi_0}^{2\pi+\phi_0}\left[1 - \frac{\beta'(\mathrm{C}'')^2}{2} \mp \beta(\mathrm{C}')\beta'(\mathrm{C}'')\cos\psi\right]d\phi \\ &= \frac{2\pi R}{v'_{\mathrm{A}}}\left[1 - \frac{\beta'(\mathrm{C}'')^2}{2} \mp \beta(\mathrm{C}')\beta'(\mathrm{C}'')\cos\psi\right] \end{aligned} \tag{2.10}$$

and ψ is given, in terms of θ_0 and λ_0, by (2.9) with $0 < \theta_0 < \pi/2$. Note that the value of the ϕ integral in (2.10) is independent of ϕ_0. The duration, T', of a Great Circle flight, as recorded by C$'$, is independent of the value of ψ:

$$T' = \frac{2\pi R}{v'_{\mathrm{A}}}. \tag{2.11}$$

The difference of the time intervals recorded by C$''$ and C$'$, $\Delta T'$ is then, from (2.10) and (2.11):

$$\Delta T'(\theta_0, \lambda_0, \pm) \equiv T''(\theta_0, \lambda_0, \pm) - T' = T'\left[-\frac{\beta'(\mathrm{C}'')^2}{2} \mp \beta(\mathrm{C}')\beta'(\mathrm{C}'')\cos\psi\right]. \tag{2.12}$$

It follows from (2.12) that:

$$\begin{aligned} \Delta T''(\theta_0, \lambda_0, +-) &\equiv \Delta T'(\theta_0, \lambda_0, +) - \Delta T'(\theta_0, \lambda_0, -) \\ &= -2T'\beta(\mathrm{C}')\beta'(\mathrm{C}'')\cos\psi = -\frac{4\pi R v_{\mathrm{E}}\cos\theta_0\cos\lambda_0}{c^2}. \end{aligned} \tag{2.13}$$

Notice that, in this approximation where terms of $\mathrm{O}(\beta^4)$ and higher are neglected, the value of $\Delta T''(\theta_0, \lambda_0, +-)$ depends only on c, the radius of the Earth and the values of ψ and v_{E}, *being independent of the speed of the aircraft, v'_{A} in the frame S'.*

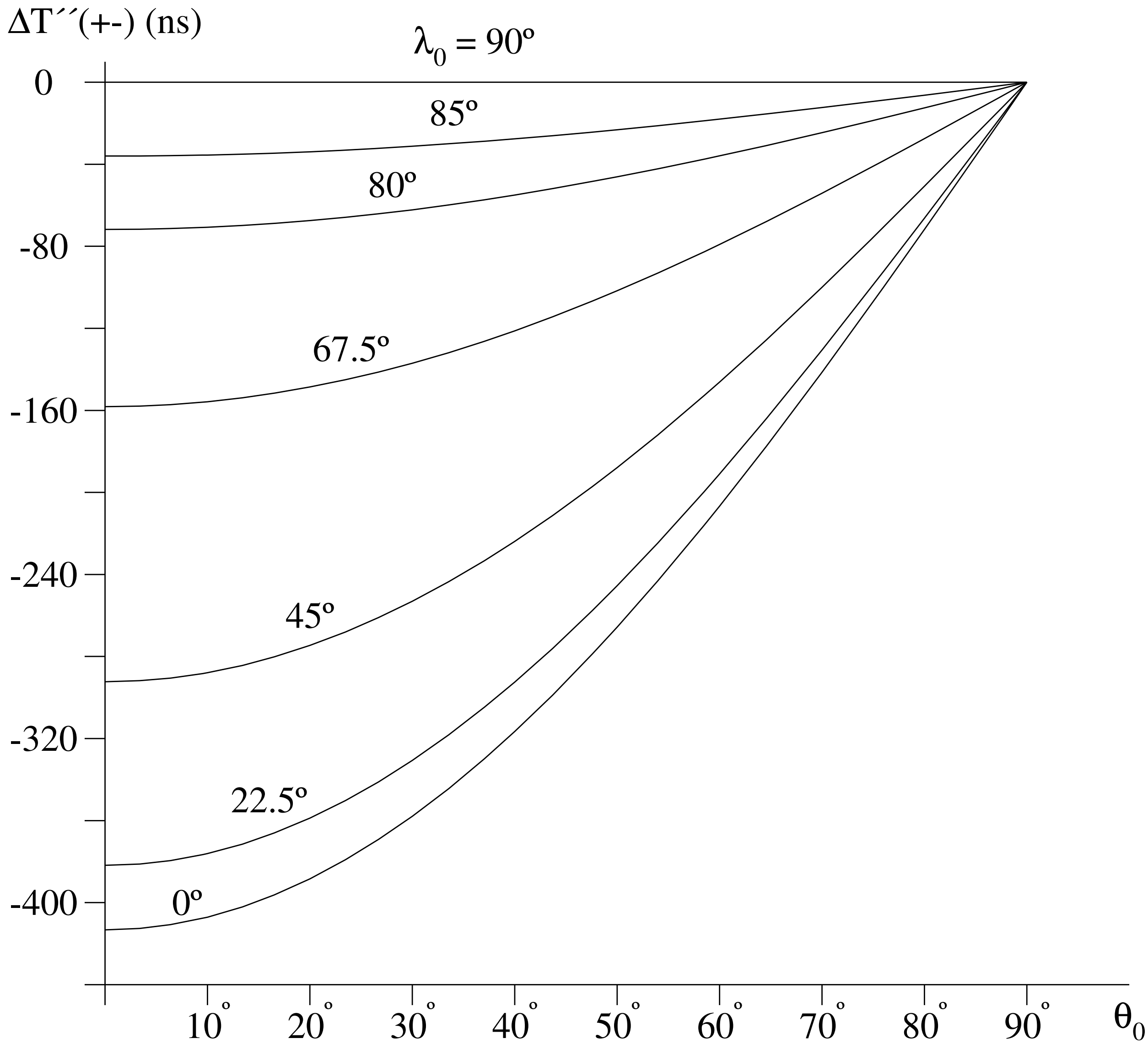

Figure 3: $\Delta T''(+-) \equiv \Delta T'(\mathrm{W}-\mathrm{E}) - \Delta T'(\mathrm{E}-\mathrm{W})$ *(in ns) as a function of* θ_0 *for fixed values of the latitude angle* λ_0 *of the point P in Fig. 1 at which the Great Circle flight starts. Note that the curves are independent of the value of* v'_{A}. $R = 6.38 \times 10^6 m$ *and* $v_{\mathrm{E}} = 464 m/s$.

Curves of $\Delta T''(\theta_0, \lambda_0, +-)$ as a function of θ_0 for various values of λ_0 are shown in Fig. 3 where, in (2.13), $R = 6.38 \times 10^6$m and $v_{\mathrm{E}} = 464$m/s. As just mentioned, the curves in Fig. 3 are independent of the value of v'_{A}. The radius of the Earth is determined from (2.11) by the observed value of T', from the known value of v'_{A}. The direction of the line of fixed longitude at P is determined by the orientation of the Great Circle for which $\Delta T''(\theta_0, \lambda_0, +-)$ vanishes ($\theta_0 = 90°$ in Fig. 3). The line of fixed latitude at P is perpendicular to this direction. The direction of rotation of the Earth is determined by the sign of $\Delta T''(\theta_0 = 0, \lambda_0, +-)$. If it is positive (negative) the aircraft moves in the E–W (W–E) direction. To now determine the value of $v_{\mathrm{E}}(\lambda = 0)$, and hence $|\vec{\Omega}| = v_{\mathrm{E}}(\lambda = 0)/R$, the clock C$'$ is transported along the line of longitude in the direction of increasing $|\Delta T''(\theta_0 = 0, \lambda_0, +-)|$ until the maximum value of this quantity is obtained. The corresponding Great Circle is the Equator ($\theta_0 = \lambda_0 = \psi = 0$). It then follows from (2.13) that:

$$|\vec{\Omega}| = \frac{v_{\mathrm{E}}}{R} = c^2 \frac{|\Delta T''(\theta_0 = 0, \lambda_0 = 0, +-)|}{4\pi R^2}. \tag{2.14}$$

The radius of the Earth and the position of the Equator, as well as the positions of the North and South Poles (i.e the direction of the axis of rotation of the Earth), are now known. Since the direction of rotation along lines of constant latitude, as well as the value of $|\vec{\Omega}|$, are also known, $\vec{\Omega}$ is determined and the problem posed above is solved.

Inclusion of GR effects results in a correction factor $1 + 2\phi_{\mathrm{E}}/c^2$ on the right side of (2.14) (see Eq. (B.11)) where $\phi_{\mathrm{E}} = -GM_{\mathrm{E}}/R = -6.25 \times 10^{11}(\mathrm{m/s})^2$, so that the correction factor has the value $1 - 1.38 \times 10^{-5}$, to be compared with a relative experimental error of about 15% on the measurement of $\Delta T''(+-)$ in the HKE experiment.

It has become customary in the literature [20, 21] to call the formula (2.14) which embodies the asymmetric time dilation effect observed in the HKE experiment, a 'Sagnac Effect' due to the accidental mathematical identity, at lowest order in β, of the equation with that giving the difference of shifts of arrival times of light signals, travelling around the Earth along its Equator, in the W$-$E or E$-$W directions, due to the Earth's easterly rotation. This is a purely classical O(β) effect, a consequence of differing relative velocities of the light signal (which moves with speed c in the ECI frame) and the receiver on the surface of the Earth. In contrast Eq. (2.14) arises from the purely relativistic (O($\geq \beta^2$)) TD effect. The incorrect conflation of the classical Sagnac effect [22, 23] with the relativistic TD effect for moving clocks has also recently been pointed out by Gezari [24].

It is interesting to note that recent experiments [25, 26] have shown that the Sagnac effect can be used, not only to detect rotation, as in ring laser and fibre optic gyroscopes, but also uniform translational motion of a light-carrying fibre optic cable.

3 Internal measurement of the speed of motion of an arbitrary point on the Earth's surface using a moveable clock

Over limited regions of the Earth's surface, the latitude-dependent velocity $v_{\mathrm{E}}(\lambda)$ may be considered constant. For example, for displacements of $\pm$100km along a line of longitude, near $\lambda = 45°$ the value of $v_{\mathrm{E}}(\lambda)$ changes by only $\pm 1.6\%$. The average value of $v_{\mathrm{E}}(\lambda)$ in such a restricted region can be measured locally by taking advantage of the same frame-dependent time dilation effects as manifested in the HKE.

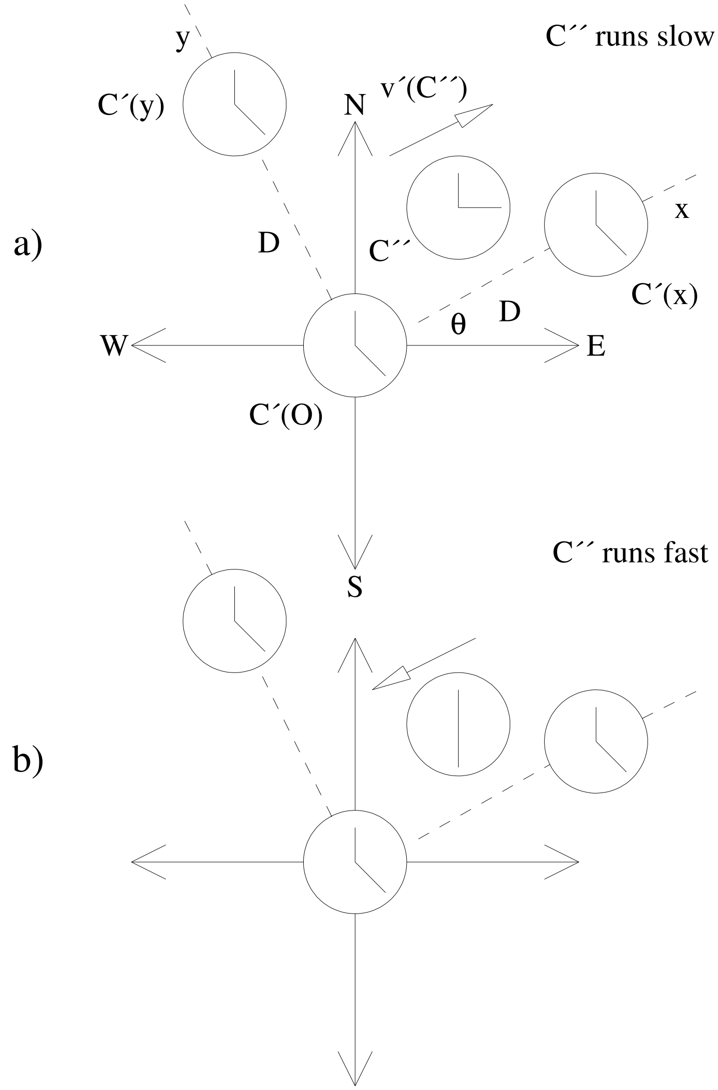

Figure 4: *Arrangement of clocks for a local measurement of the velocity, in the frame S, of a point on the Earth's surface.* C$'$(O), C$'(x)$ *and* C$'(y)$ *are synchronised clocks, at fixed locations, between which the clock* C$''$ *is moved at constant speed. In a) where* C$''$ *moves along the positive x-axis it is seen by observers at rest on the Earth's surface to run slow relative to the clocks* C$'$*. In b) where* C$''$ *moves in the negative $x-$direction, at the same speed, it is seen to run fast.*

An experimental setup for such a measurement is shown schematically in Figs. 4 and 5. The principle of the measurement is illustrated in Fig. 4. Clocks C'(O), C'(x) and C'(y) are placed, respectively, at the origin and at equal distances, D, along the $x-$ and $y-$axes of an arbitrarily-oriented Cartesian coordinate system on the surface of the Earth. The above three clocks are synchronised, and a fourth clock, C$''$, moves along the $x-$ or $y-$axes at constant speed v'. Some different methods of synchronising spatially-separated

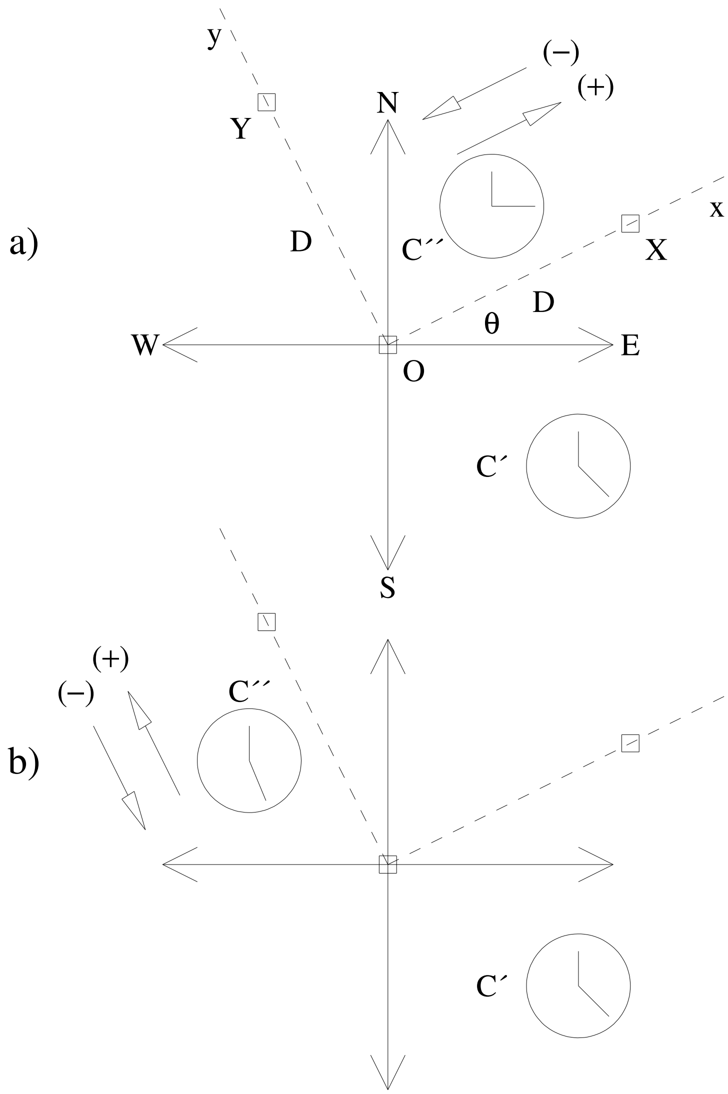

Figure 5: *a) Measurement of $\Delta T''(x,+-)$ by moving C″ along the x-axis. a) Measurement of $\Delta T''(y,+-)$ by moving C″ along the y-axis. Synchronised clocks as shown in Fig. 4. are located at the marker positions: O, X, Y. See text for discussion.*

clocks, without the use of light signals, are described in [27]. A convenient method for the experiment described here, as well as that of Section 5, is to use a signal cable[5] with a known delay time t_D. The distant to-be-synchronised clock is stopped and set to show the time t_D. The local clock is started, with initial setting zero, at the instant that the synchronisation signal is sent along the cable. The distant clock, started on reception of the signal, is then synchronised with the local clock. It is important to remark that the 'slow clock transport method' [28] cannot be used to synchronise separated clocks on the surface of the Earth unless they lie on the same line of longitude ($\theta = 90°$ in Eq. (2.7)). The time intervals recorded by C$''$ as it moves from the position of clock C'(O) to that of C'(x), $T''(x,+)$, or *vice versa*, $T''(x,-)$, are compared with those recorded by C'(O) and C'(x). Similar comparisons are made for the intervals $T''(y,+)$ and $T''(y,-)$ which are similarly defined for motion along the $y-$axis. As shown in Appendix A, the geometry of Fig. 4 and (2.7) gives, on retaining only O(β^2) terms on the right side of (2.7):

$$\Delta T'(x,+) \equiv T''(x,+) - T' = \frac{D}{v'}\left[-\frac{1}{2}\left(\frac{v'}{c}\right)^2 - \frac{v' v_{\rm E}(\lambda)}{c^2}\cos\theta\right], \quad (3.1)$$

$$\Delta T'(x,-) \equiv T''(x,-) - T' = \frac{D}{v'}\left[-\frac{1}{2}\left(\frac{v'}{c}\right)^2 + \frac{v' v_{\rm E}(\lambda)}{c^2}\cos\theta\right], \quad (3.2)$$

$$\Delta T'(y,+) \equiv T''(y,+) - T' = \frac{D}{v'}\left[-\frac{1}{2}\left(\frac{v'}{c}\right)^2 + \frac{v' v_{\rm E}(\lambda)}{c^2}\sin\theta\right], \quad (3.3)$$

$$\Delta T'(y,-) \equiv T''(y,-) - T' = \frac{D}{v'}\left[-\frac{1}{2}\left(\frac{v'}{c}\right)^2 - \frac{v' v_{\rm E}(\lambda)}{c^2}\sin\theta\right]. \quad (3.4)$$

where $T' = D/v'$ is the time interval recorded by C'(O), C'(x) and C'(y) during the passages of C$''$. Eq. (3.1) shows that $T''(x,+) - T' < 0$ for $\cos\theta > 0$, so that, as shown in Fig. 4a, for motion in the positive $x-$direction, C$''$ is seen to run slow relative to clocks at rest on the surface of the Earth. In the case that $v_{\rm E}(\lambda)\cos\theta > v'/2$, (3.2) gives $T''(x,-) - T' > 0$ and, as shown in Fig. 4b, for motion in the negative $x-$direction, C$''$ runs *fast* relative to such clocks. Fig. 4 shows clearly that the observed TD effect in the frame S' does not depend only on the relative velocity of the stationary and moving clocks in the experimemt, as might naively be expected if 'everything is relative'. Indeed, in Fig. 4b there is a 'time contraction', not a time dilation effect, as the moving clock actually runs faster than the stationary ones. This demonstrates the fundamental importance of the concepts of 'base' and 'travelling' frames [5, 6], or the equivalent one of 'coordinate time' [3, 4]) (as recorded by a hypothetical clock at rest in the frame in which the velocity of a moving clock is specified) for the correct description of temporal effects in space-time experiments.

It follows from (3.1)–(3.4) that:

$$\Delta T'(x,+) - \Delta T'(x,-) \equiv \Delta T''(x,+-) = -\frac{2Dv_{\rm E}(\lambda)}{c^2}\cos\theta, \quad (3.5)$$

$$\Delta T'(y,+) - \Delta T'(y,-) \equiv \Delta T''(y,+-) = \frac{2Dv_{\rm E}(\lambda)}{c^2}\sin\theta. \quad (3.6)$$

[5]This should be a conventional cable with a metallic conductor in which the signal is carried by conduction electrons. The light signal propagation time in a fiber-optic cable depends on the linear motion of the cable due to the Sagnac effect [25, 26].

Notice that although the time interval T' formally cancels in the definition of $\Delta T''$, it is crucial for the experiment that what can be observed with high precision —as done in the HKE— is only the *difference* between the time intervals recorded by C" and C'. This is given by comparing the readings of the two clocks, which, if they run at the same constant rate, can, in principle, be done at any time after C" is brought to rest in the frame S'. It is clearly practically impossible to measure the tiny difference between, say, $T''(x,+)$ and $T''(x,-)$ by attempting to measure these intervals simply by observing the epochs recorded by C" at the beginning and end of the transit. However, if this could be done, the clock C' would actually not be needed to determine $\Delta T''(+-)$, and the experiment could be performed using the clock C" alone, thus avoiding completely the necessity of clock synchronisation..

The measurements of $\Delta T''(x,+-)$ and $\Delta T''(y,+-)$ now give the magnitude of the local velocity of the surface of the Earth, as well as the direction of the local line of fixed latitude:

$$
\begin{aligned}
v_{\mathrm{E}}(\lambda) &= \frac{c^2}{2D}[\Delta T''(x,+-)^2 + \Delta T''(y,+-)^2]^{\frac{1}{2}}, && (3.7)\\
\cos\theta &= \frac{-\Delta T''(x,+-)}{[\Delta T''(x,+-)^2 + \Delta T''(y,+-)^2]^{\frac{1}{2}}}, && (3.8)\\
\sin\theta &= \frac{\Delta T''(y,+-)}{[\Delta T''(x,+-)^2 + \Delta T''(y,+-)^2]^{\frac{1}{2}}}. && (3.9)
\end{aligned}
$$

A scheme for an actual experiment is shown in Fig. 5. In some flat deserted region of the Earth, markers O, X and Y, each equipped with a synchronised clock, are set up at the origin and at equal distances, D, along the axes of a Cartesian coordinate system. The precision clock C'' is placed in a helicopter and flown, at constant speed, from O to X then from X to O (Fig. 5a) and from O to Y then from Y to O (Fig. 5b). During the passages the four time interval differences $T''(x,\pm)-T'$ and $T''(y,\pm)-T'$ are measured. Performing the experiment at the Equator where $v_{\mathrm{E}}(\lambda=0) = 464$m/s, and setting $D = 100$km, gives $\Delta T''(x,+-) = 1.04$ns for $\theta = 0$. If the speed of the helicopter is 300km/h the time intervals T'' must therefore be measured with a relative precision considerably better than $\simeq 10^{-11}$ over the flight time $T' = 20$m. This is well within the capability of modern atomic clocks which can measure time intervals with relative precision of 10^{-15}/day [29].

4 Internal measurement using moveable clocks of the velocity $\vec{v}$ of an arbitrary travelling frame in free space

The comoving frames S' and S" of the clocks C' and C'' during the measurements described in the last section are not inertial frames, as they are subject to transverse acceleration. However as previously discussed and experimentally verified, the TD effects are the same as they would be in strictly inertial frames moving with the same total velocity relative to the frame S, in which coordinate time is defined. It is then evident that if the markers O, X and Y shown in Fig. 5 were situated in free space and all moved with the same uniform velocity in the x–y plane in S, the predictions of Eqs. (3.7)-(3.9) would be unchanged. Thus the velocity of an inertial travelling frame S', relative to its base frame S, can, in general, be determined by purely internal measurements: observations of $\Delta T''$, in S'.

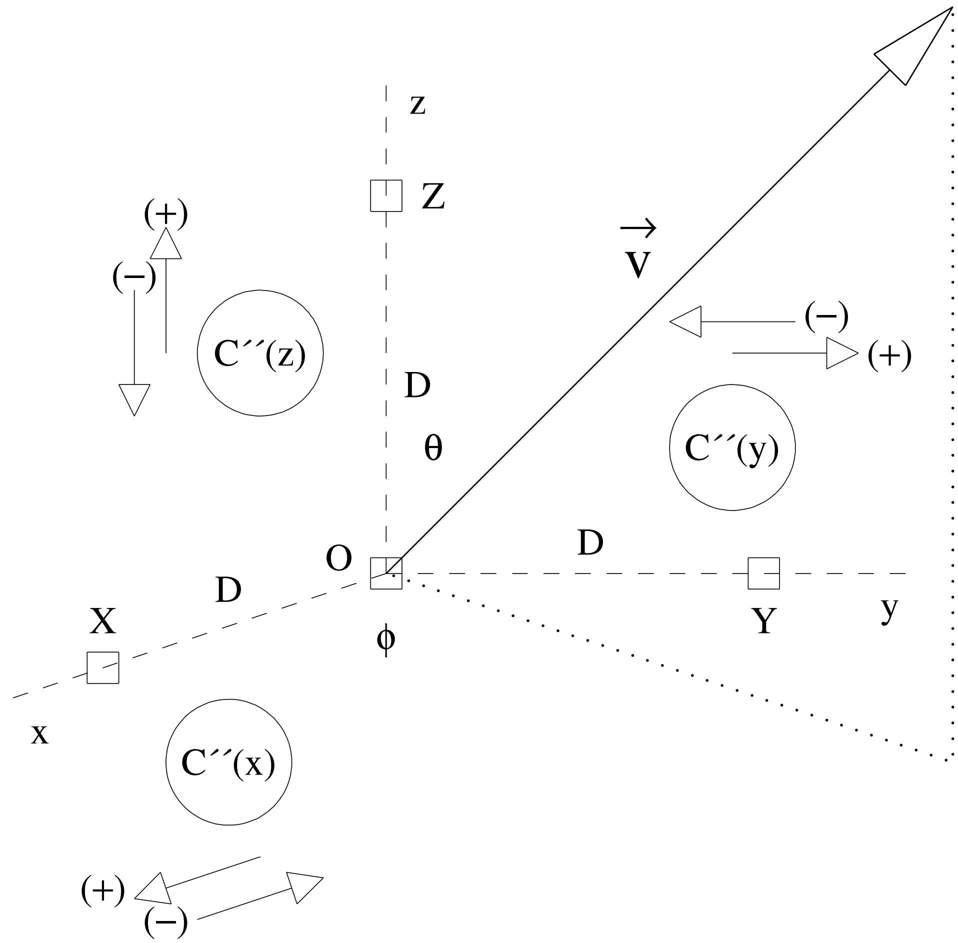

Figure 6: *Clock displacements to measure the time intervals* $\Delta T''(\chi, +-)$ $(\chi = x, y, z)$ *from which the velocity of the travelling inertial frame S' relative to inertial base frame S,* $\vec{v}$*, may be determined. Synchronised clocks (not shown) are located at the marker positions: O, X, Y and Z. See text for discussion.*

The measurement of the magnitude and direction, in a plane, of the velocity of S' relative to S given by Eqs. (3.7)-(3.9) is readily generalised to motion in three spatial dimensions. How this might be done is shown schematically in Fig. 6. The marker objects O,X,Y and Z are situated, respectively, at the origin and at equal distances, D, along arbitrarily oriented $x-$, $y-$ and $z-$axes of a Cartesian coordinate system in free space. Each marker object is equipped with a synchronised clock: C$'$(O), C$''$(X), C$'$(Y) and C$'$(Z). The clocks may conveniently be synchronised by the method described in the previous section, with light signals in free space replacing the cable-borne ones. The clocks at X, Y and Z are stopped and set to $t' = D/c$. At time $t' = 0$ the clock at O is started and light signals are sent towards the other clocks. The latter start on receiving the light signals. All four clocks are then synchronised. When the clocks are simultaneously accelererated, in the base frame, up to the velocity $\vec{v}$ they remain synchronised when observed in this frame [30]. The base frame S is the one in which the velocity $\vec{v}$ of the travelling frame S' is specified (an initial condition of the experiment). Coordinate time is recorded by clocks at rest in S. Since all time intervals in the frames S' and S" are calculated in terms of coordinate time intervals, Δt, using the time dilation relations (1.3) there is no possiblity that any 'relativity of simultaneity' effects defined as frame-dependent differences of *clock settings* of separate clocks, not differences of *time intervals* recorded by such clocks, can play any role in the calculations. Notice that, as mentioned in the previous section, the clocks at O, X, Y and Z are only placed, for experimental convenience, to provide direct measurements of $\Delta T'(+)$ and $\Delta T'(-)$. If the time intervals $T''(+)$ and $T''(-)$ are directly observed only the single (moving) clock C" would suffice to perform the whole experiment and no synchronisation of clocks at rest in the frame S' is required.

The magnitude and direction of the relative velocity, $\vec{v}$, of S' and S is then internally determined, during the phase of uniform motion, by moving the clocks C$''(x)$, C$''(y)$ and C$''(z)$, between the marker objects, at constant speed along the coordinate axes as shown in Fig. 6, and observing the time interval difference $\Delta T''$ between this clock and the synchronised fixed clocks at the marker positions. Of course, the same, physical, moving clock can be used to perform all the measurements. The geometry of Fig. 6 gives, as the three-dimensional generalisation of Eqs. (3.5) and (3.6):

$$\Delta T''(z,+-) = -\frac{2Dv}{c^2}\cos\theta, \tag{4.1}$$

$$\Delta T''(y,+-) = -\frac{2Dv}{c^2}\sin\theta\sin\phi. \tag{4.2}$$

$$\Delta T''(x,+-) = -\frac{2Dv}{c^2}\sin\theta\cos\phi. \tag{4.3}$$

so that (3.7)-(3.9) generalise to

$$v = \frac{c^2}{2D}[\Delta T''(x,+-)^2 + \Delta T''(y,+-)^2 + \Delta T''(z,+-)^2]^{\frac{1}{2}}, \tag{4.4}$$

$$\cos\theta = -\frac{\Delta T''(z,+-)}{[\Delta T''(x,+-)^2 + \Delta T''(y,+-)^2 + \Delta T''(z,+-)^2]^{\frac{1}{2}}}, \tag{4.5}$$

$$\sin\phi = -\frac{\Delta T''(y,+-)}{[\Delta T''(x,+-)^2 + \Delta T''(y,+-)^2]^{\frac{1}{2}}}, \tag{4.6}$$

$$\cos\phi = -\frac{\Delta T''(x,+-)}{[\Delta T''(x,+-)^2 + \Delta T''(y,+-)^2]^{\frac{1}{2}}}. \tag{4.7}$$

For velocities $v \simeq v_{\rm E} = 464$m/s and distances D of the order of the circumference of the Earth $\simeq 4 \times 10^4$km, the values of $\Delta T''$ will be similar than those shown in Fig. 3 —up to several hundred nanoseconds— and so easily measurable by currently available atomic clocks.

5 Internal measurement using a moveable clock of the speed of Galileo's ship

A common way of stating the Special Relativity Principle (SRP) is to assert the impossibility, by any physical means whatever, to detect the existence of uniform translational motion by purely internal measurements, This was done in a particularly graphic way by Galileo in his book 'Dialogue Concerning the Two Chief World Systems —Ptolemaic and Copernican' [31] in the following (abridged) passage:

> 'Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speeds to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; . . . When you have observed all these things carefully . . . have the ship proceed with any speed you like, so long as the speed is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. . . . The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in the water will swim toward the front of their bowl with no more effort than toward the back and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally butterflies and flies will continue their flights indifferently toward every side, nor will it happen that they are concentrated toward the stern, as if tired from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air'.

The assertion of this passage that the laws of physics are the same in different inertial frames is a valid one for mechanical processes —indeed it follows as a necessary consequence of the invariance of Newton's Second Law of Motion under the Galilean transformations: $x' = x - vt$, $t' = t$. However, as will now be demonstrated, it is sufficent to have synchronised precision clocks at different positions in the ship, together with another one that can be moved about the ship at constant speed, to measure, internally, the speed of the ship relative to the surface of the Earth, as well as the local speed of motion of the surface of the Earth in the inertial frame S. As in the previous examples, if time intervals registered by the moving clock during its displacements were directly observable, this clock alone would suffice to perform the experiment.

A precision clock, C$'''$, with proper frame S$'''$, recording time t''', moveable at constant speed v'' in the proper frame S" of the ship, that may be compared with synchronised clocks C$''(i)$, recording time t'', at different positions, i, in the ship, is introduced. It is assumed that the ship always moves parallel to a line joining the stern of the ship to its bow, and that C$'''$ is displaced along this same line, where it is compared with clocks C$''(1)$ and C$''(2)$ which are a distance D apart, say at the stern and bow, respectively, of the ship. The calculation of the speed v' of the ship (which is assumed to move at an angle θ relative to the local line of fixed latitude) in the local comoving rest frame S' of the Earth's surface, is a straightfoward generalisation of the one presented in Section 3 above. The TD effect for the clocks C$''$ and C$'''$ is given (see Eq. (1.3)) by the equations:

$$dt = \gamma(\mathrm{C}'')dt'' = \gamma(\mathrm{C}''')dt'''. \tag{5.1}$$

Analogously to Eq. (2.5):

$$\gamma(\mathrm{C}''',\pm) = \gamma(\mathrm{C}')[\gamma'(\mathrm{C}''',\pm) + \beta(\mathrm{C}')\beta'(\mathrm{C}''',\pm)\gamma'(\mathrm{C}''',\pm)\cos\theta] \tag{5.2}$$

where

$$\gamma'(\mathrm{C}''',\pm) = \gamma'(\mathrm{C}'')[\gamma''(\mathrm{C}''') \pm \beta'(\mathrm{C}'')\beta''(\mathrm{C}''')\gamma''(\mathrm{C}''')] \tag{5.3}$$

and

$$\beta'(\mathrm{C}''',\pm)\gamma'(\mathrm{C}''',\pm) = \gamma'(\mathrm{C}'')[\pm\beta''(\mathrm{C}''')\gamma''(\mathrm{C}''') + \beta'(\mathrm{C}'')\gamma''(\mathrm{C}''')] \tag{5.4}$$

with the $+$ $(-)$ signs corresponding to motion of C$'''$ from stern to bow (bow to stern). Using (5.3) and (5.4) to eliminate $\gamma'(\mathrm{C}''',\pm)$ and $\beta'(\mathrm{C}''',\pm)\gamma'(\mathrm{C}''',\pm)$ from (5.2) gives:

$$\gamma(\mathrm{C}''',\pm) = \gamma(\mathrm{C}')\gamma'(\mathrm{C}'')\gamma''(\mathrm{C}''')\{1 \pm \beta'(\mathrm{C}'')\beta''(\mathrm{C}''') + \beta(\mathrm{C}')[\beta'(\mathrm{C}'') \pm \beta''(\mathrm{C}''')]\cos\theta\} \tag{5.5}$$

Combining this equation with (2.5) and (5.1):

$$dt'''(\pm) = \frac{\gamma(\mathrm{C}'')dt''}{\gamma(\mathrm{C}''',\pm)} = \frac{(1 + \beta(\mathrm{C}')\beta'(\mathrm{C}'')\cos\theta)dt''}{\gamma''(\mathrm{C}''')\{1 \pm \beta'(\mathrm{C}'')\beta''(\mathrm{C}''') + \beta(\mathrm{C}')[\beta'(\mathrm{C}'') \pm \beta''(\mathrm{C}''')]\cos\theta\}} \tag{5.6}$$

If $T'''(\theta,+)$ and $T'''(\theta,-)$ are the time intervals recorded by C$'''$ for passages from stern to bow and from bow to stern respectively, then since the corresponding time interval T'' recorded by C$''(1)$ or C$''(2)$ is D/v'', (5.6) gives, on retaining only O(β^2) terms, the equations analogous to (3.1)–(3.4):

$$\Delta T''(\theta,+) \equiv T'''(\theta,+) - T'' = \frac{D}{v''}\left[-\frac{1}{2}\left(\frac{v''}{c}\right)^2 - \frac{v''(v' + v\cos\theta)}{c^2}\right], \tag{5.7}$$

$$\Delta T''(\theta,-) \equiv T'''(\theta,-) - T'' = \frac{D}{v''}\left[-\frac{1}{2}\left(\frac{v''}{c}\right)^2 + \frac{v''(v' + v\cos\theta)}{c^2}\right] \tag{5.8}$$

from which follows, analogously to (3.5) or (3.6):

$$\Delta T'''(\theta,+-) \equiv \Delta T''(\theta,+) - \Delta T''(\theta,-) = -\frac{2D}{c^2}(v' + v\cos\theta) \tag{5.9}$$

If $v' = 0$ then (3.5) is recovered from (5.9) and $\Delta T'''$ measures $v\cos\theta$ so that if θ is known v may be determined. Note that $\Delta T'''$ is independent of v'', the speed of the moving clock C'''. In order to measure v' directly from (5.9) it is necessary that $\theta = 90^\circ$ so that it is known that the ship moves directly N$-$S, or S$-$N, i.e. along a line of fixed longitude.

A procedure to measure all three unknown quantities v', v and θ in (5.9) is to perform three separate measurements of $\Delta T'''$ first according to (5.9) and then on changing the direction of motion of the ship by by two successive anti-clockwise rotations of 120° each. This gives the following three relations:

$$\Delta T'''(1) \equiv \Delta T'''(\theta,+-) = -\frac{2D}{c^2}(v' + v\cos\theta), \tag{5.10}$$

$$\Delta T'''(2) \equiv \Delta T'''(\theta+120^\circ,+-) = -\frac{2D}{c^2}(v' - vS\cos\theta - vC\sin\theta), \tag{5.11}$$

$$\Delta T'''(3) \equiv \Delta T'''(\theta+240^\circ,+-) = -\frac{2D}{c^2}(v' - vS\cos\theta + vC\sin\theta) \tag{5.12}$$

where

$$S \equiv \sin 30^\circ = 1/2, \qquad C \equiv \cos 30^\circ = \sqrt{3}/2.$$

These simultaneous equations may be solved to give v', v and θ. It is found that:

$$v' = -\frac{c^2[2S\Delta T'''(1) + \Delta T'''(2) + \Delta T'''(3)]}{4D(1+S)}, \tag{5.13}$$

$$v = \sqrt{v_S^2 + v_C^2}, \tag{5.14}$$

$$v_S \equiv v\sin\theta = -\frac{c^2[\Delta T'''(3) - \Delta T'''(2)]}{4DC}, \tag{5.15}$$

$$v_C \equiv v\cos\theta = -\frac{c^2[2\Delta T'''(1) - \Delta T'''(2) - \Delta T'''(3)]}{4D(1+S)}. \tag{5.16}$$

If the direction of motion of the ship relative to the local line of fixed latitude is known, two measurements of $\Delta T'''$ suffice to determine both v' and v, provided the ship does not move along a line of fixed longitude. For example, for West to East and East to West motion, corresponding to a change of 180° in the direction of motion of the ship, (5.9) gives:

$$\Delta T'''(\mathrm{W}-\mathrm{E},+-) = -\frac{2D}{c^2}(v' + v), \tag{5.17}$$

$$\Delta T'''(\mathrm{E}-\mathrm{W},+-) = -\frac{2D}{c^2}(v' - v). \tag{5.18}$$

So that

$$v' = -\frac{c^2[\Delta T'''(\mathrm{W}-\mathrm{E},+-) + \Delta T'''(\mathrm{E}-\mathrm{W},+-)]}{4D}, \tag{5.19}$$

$$v = -\frac{c^2[\Delta T'''(\mathrm{W}-\mathrm{E},+-) - \Delta T'''(\mathrm{E}-\mathrm{W},+-)]}{4D}. \tag{5.20}$$

Assuming $D = 200$m and equatorial motion so that $v = 464$m/s, it is found that, since $v' \ll v$, that

$$-\Delta T'''(\mathrm{W}-\mathrm{E},+-) \simeq \Delta T'''(\mathrm{E}-\mathrm{W},+-) \simeq \ 2ps$$

If the clock C$'''$ is moved at walking pace (5 km/h) from the stern of the ship to the bow and *vice versa* the time recorded by C$''$(1) and C$''$(2) during a single clock transport is 144s. The corresponding time uncertainty for time measurement by a clock with the current stability of 100ps/day is 0.17ps, which gives an 8.5% fractional error in the measurement of $\Delta T'''$ or of v. If $v' = 60$km/h (16.7m/s) then $v'/v = 0.035$. Therefore a one order of magnitude improvement in clock stability would be required to detect v' at the four standard deviation level, and two orders of magnitude improvement to measure v' with a relative precision of 0.2%. Such an experiment, which seems within the bounds of experimental possibility, would then falsify, in a convincing manner, the application in the real world of the SRP as a statement of the impossibility, by any physical means whatever, to detect by purely internal measurements, the existence of uniform translational motion. Because, however, Newton's Second Law of motion, in the form $d\vec{p}/dt = \vec{F}$ with suitably defined relativistic definitions of momentum, $\vec{p}$, and force, $\vec{F}$, holds in all inertial frames, the SRP retains its validity in special relativity for all *mechanical* experiments, and it remains true that the results of such experiments are unable to detect, internally, the presence of uniform translational motion. Galileo's assertion concerning mechanical experiments therefore remains valid in special relativity. A 'mechanical experiment' is defined in the present paper as one in which Newton's Second Law, or some theoretically equivalent concept, such as Hamilton's Principle, is essential for its analysis. No such laws are invoked in the purely space-time geometric derivation of the asymmetric time dilation relations (1.3).

Appendix A

The unit vector $\hat{t}$, tangent to the circle of fixed latitude, in the W$-$E direction, at any point on the Great Circle followed by the aircraft (see Figs. 1 and 2), can be written in primed coordinates as:

$$\hat{t}_{x'} = -\sin\phi', \tag{A.1}$$
$$\hat{t}_{y'} = \cos\phi', \tag{A.2}$$
$$\hat{t}_{z'} = 0 \tag{A.3}$$

where ϕ' is the azimuthal angle measured from the line segment QN, (see Fig. 1) in the plane of a circle of fixed latitude λ.

Transforming $\hat{t}$ into the unprimed system according to:

$$x = x', \tag{A.4}$$
$$y = y'\cos\psi + z'\sin\psi, \tag{A.5}$$
$$z = z'\cos\psi - y'\sin\psi \tag{A.6}$$

gives

$$\hat{t}_x = -\sin\phi', \tag{A.7}$$
$$\hat{t}_y = \cos\phi'\cos\psi, \tag{A.8}$$
$$\hat{t}_z = -\cos\phi'\sin\psi. \tag{A.9}$$

Since the projection of $\hat{t}$ into the xy plane (the plane of the Great Circle around which the aircraft moves) is tangent to the Great Circle at P (see Fig. 2a):

$$\frac{\hat{t}_x}{\hat{t}_y} = -\tan\phi = -\frac{\tan\phi'}{\cos\psi} \tag{A.10}$$

so that

$$\tan\phi' = \cos\psi\tan\phi. \tag{A.11}$$

Using (A.11) to eliminate ϕ' from (A.7)-(A.9) gives:

$$\hat{t}_x = -\frac{\cos\psi\tan\phi}{(1+\cos^2\psi\tan^2\phi)^{\frac{1}{2}}}, \tag{A.12}$$

$$\hat{t}_y = \frac{\cos\psi}{(1+\cos^2\psi\tan^2\phi)^{\frac{1}{2}}}, \tag{A.13}$$

$$\hat{t}_z = -\frac{\sin\psi}{(1+\cos^2\psi\tan^2\phi)^{\frac{1}{2}}}. \tag{A.14}$$

Since

$$(\vec{v}'_A)_x = -v'_A\sin\phi, \tag{A.15}$$

$$(\vec{v}'_A)_y = v'_A\cos\phi, \tag{A.16}$$

$$(\vec{v}'_A)_z = 0. \tag{A.17}$$

it follows that

$$\cos\theta = \frac{\hat{t}\cdot\vec{v}'_A}{|\vec{v}'_A|} = \frac{\cos\psi\tan\phi\sin\phi+\cos\psi\cos\phi}{(1+\cos^2\psi\tan^2\phi)^{\frac{1}{2}}} = \frac{\cos\psi}{(1-\sin^2\psi\sin^2\phi)^{\frac{1}{2}}} \tag{A.18}$$

which is Eq. (2.1) of the main text. Eq. (2.2) generalises, for an arbitrary point on the Great Circle, to:

$$\sin\lambda = \sin\psi\sin\phi. \tag{A.19}$$

Eliminating ϕ between (A.18) and (A.19) gives:

$$\cos\psi = \cos\theta\cos\lambda \tag{A.20}$$

which is the generalisation of Eq. (2.9) of the text for an arbitrary point on the Great Circle.

It follows from (2.4) that:

$$\beta(\mathrm{C}',\lambda)\cos\theta = \beta(\mathrm{C}')\cos\lambda\cos\theta = \frac{\beta(\mathrm{C}')(1-\sin^2\psi\sin^2\phi)^{\frac{1}{2}}\cos\psi}{(1-\sin^2\psi\sin^2\phi)^{\frac{1}{2}}} = \beta(\mathrm{C}')\cos\psi \tag{A.21}$$

where (A.18) and (A.19) are used to eliminate θ and λ respectively.

Setting $v'_{\mathrm{A}} = v'$, then (2.7) is written explicitly in terms of v' and $v_{\mathrm{E}}(\lambda)$ as:

$$dt'' = \frac{[\sqrt{1-(v'/c)^2}]dt'}{\left[1+\frac{v'v_{\mathrm{E}}(\lambda)}{c^2}\cos\theta\right]} \tag{A.22}$$

Retaining only $O(\beta^2)$ terms on the right side of this equation gives:

$$dt'' = \left[1 - \frac{1}{2}\left(\frac{v'}{c}\right)^2 - \frac{v' v_{\mathrm{E}}(\lambda)}{c^2}\cos\theta\right] dt' \tag{A.23}$$

The formula (3.1), for example, is derived from (A.23) on making the replacements $dt'' \to T''(x,+)$, $dt' \to T' = D/v'$ in (A.23). Other appropriate replacements yield Eqs.(3.2)-(3.4).

Appendix B

Taking into account both SR and GR effects, for a clock in motion on the surface of the Earth, assumed to be spherical, of radius R, and of mass, M_{E}, an increment, $d\tau$, of the proper time of the clock is related to the corresponding increment, dt of coordinate time, as discussed in Section 2, by the Schwartzschild metric equation [32, 33]

$$d\tau = \left[\left(1 + \frac{2\phi_{\mathrm{E}}}{c^2}\right) - \frac{1}{c^2}\left(\frac{v_r^2}{1 + \frac{2\phi_{\mathrm{E}}}{c^2}} + v_\theta^2 + v_\phi^2\right)\right]^{\frac{1}{2}} dt \tag{B.1}$$

where v_r, v_θ and v_ϕ are components of the velocity of the clock in the ECI frame in a polar coordinate system with origin at the center of the Earth, polar axis oriented in the S$-$N direction and $\phi_{\mathrm{E}} = -GM_{\mathrm{E}}/R$ is the gravitational potential at the surface of the Earth. For a clock undergoing equatorial circumnavigation, $v_r = v_\theta = 0$ and $v_\phi = v_{\mathrm{E}} = c\beta_{\mathrm{E}}$. Denoting, as in the text, the proper time of a clock by t' or t'', Eq. (B.1) then simplifies, for clocks at rest on the surface of the Earth, undergoing W$-$E (+) or E$-$W ($-$) equatorial round trips at speed $v'_{\mathrm{A}} = c\beta'_{\mathrm{A}}$ relative to the surface of the Earth, to the respective equations:

$$dt' = \left[\left(1 + \frac{2\phi_{\mathrm{E}}}{c^2}\right) - \beta_{\mathrm{E}}^2\right]^{\frac{1}{2}} dt, \tag{B.2}$$

$$dt''_{\pm} = \left[\left(1 + \frac{2\phi_{\mathrm{E}}}{c^2}\right) - \beta_{\mathrm{A}}(\pm)^2\right]^{\frac{1}{2}} dt \tag{B.3}$$

where

$$\beta_{\mathrm{A}}(\pm) \equiv \frac{\beta_{\mathrm{E}} \pm \beta'_{\mathrm{A}}}{1 \pm \beta_{\mathrm{E}}\beta'_{\mathrm{A}}}. \tag{B.4}$$

Taking the quotient of (B.3) and (B.2) gives

$$dt''_{\pm} = \left[\frac{\left(1 + \frac{2\phi_{\mathrm{E}}}{c^2}\right) - \beta_{\mathrm{A}}(\pm)^2}{\left(1 + \frac{2\phi_{\mathrm{E}}}{c^2}\right) - \beta_{\mathrm{E}}^2}\right]^{\frac{1}{2}} dt'. \tag{B.5}$$

Subsituting for $\beta_{\mathrm{A}}(\pm)$ in (B.5) from (B.4) gives, after some algebraic manipulation,

$$dt''_{\pm} = \frac{\sqrt{1 - (\beta'_{\mathrm{A}})^2}}{1 \pm \beta_{\mathrm{E}}\beta'_{\mathrm{A}}}\left[\frac{1 - \beta_{\mathrm{E}}^2 + \frac{(1 \pm \beta_{\mathrm{E}}\beta'_{\mathrm{A}})^2}{1 - (\beta'_{\mathrm{A}})^2}\frac{2\phi_{\mathrm{E}}}{c^2}}{1 - \beta_{\mathrm{E}}^2 + \frac{2\phi_{\mathrm{E}}}{c^2}}\right]^{\frac{1}{2}} dt'. \tag{B.6}$$

Setting $\phi_{\rm E} = 0$, Eq. (2.7) for the case $\lambda = 0$, $\theta = 0$ or π is recovered. Retaining only the first order term in $\phi_{\rm E}$ in the power expansion of the terms in the large square bracket of (B.6) gives:

$$dt''_{\pm} = \left\{ \frac{\sqrt{1-(\beta'_{\rm A})^2}}{1 \pm \beta_{\rm E}\beta'_{\rm A}} + \frac{\phi_{\rm E}}{(1-\beta_{\rm E}^2)c^2}\left[\frac{1 \pm \beta_{\rm E}\beta'_{\rm A}}{\sqrt{1-(\beta'_{\rm A})^2}} - \frac{\sqrt{1-(\beta'_{\rm A})^2}}{1 \pm \beta_{\rm E}\beta'_{\rm A}}\right]\right\} dt'. \tag{B.7}$$

while retaining only $\mathrm{O}(\beta^2)$ terms on the right side of (B.7) gives:

$$dt''_{\pm} = \left\{1 - \frac{(\beta'_{\rm A})^2}{2} \mp \beta_{\rm E}\beta'_{\rm A} + \frac{\phi_{\rm E}}{c^2}[(1+\beta_{\rm E}^2+\beta'_{\rm A})^2 \pm 2\beta_{\rm E}\beta'_{\rm A}]\right\} dt'. \tag{B.8}$$

Integrating over the W−E or E−W round trips to give: $T''(+) \equiv \int dt''_{+}$, $T''(-) \equiv \int dt''_{-}$ and $T' \equiv \int dt'$ it is found that

$$\Delta T''(\pm) = T''(+) - T''(-) = -2T'\beta_{\rm E}\beta'_{\rm A}(1 - \frac{2\phi_{\rm E}}{c^2}) + \mathrm{O}(\phi_{\rm E}^2) \tag{B.9}$$

or, using (2.11)

$$\Delta T''(\pm) = -\frac{4\pi v_{\rm E} R}{c^2}(1 - \frac{2\phi_{\rm E}}{c^2}) + \mathrm{O}(\phi_{\rm E}^2) \tag{B.10}$$

So, on including the $\mathrm{O}(\phi_{\rm E})$ GR correction, Eq. (2.14) is modified to:

$$|\vec{\Omega}| = \frac{v_{\rm E}}{R} = c^2\frac{|\Delta T''(\theta_0 = 0, \lambda_0 = 0, +-)|}{4\pi R^2}(1 + \frac{2\phi_{\rm E}}{c^2}). \tag{B.11}$$

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