

# The Hafele-Keating experiment and internal detection of uniform translational motion

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## **Abstract**

The asymmetrical time dilation effect observed, for the first time, in the Hafele-Keating experiment allows the angular velocity of a uniformly rotating spherical object, or the velocity of an inertial frame, to be determined from purely internal observations of time interval differences between static and uniformly moving clocks. Several experimental proposals where asymmetrical time dilation is manifested are presented. The special relativistic predictions for them, the theoretical basis for which is already verified by the results of the Hafele-Keating experiment, are examples of inapplicability of the Special Relativity Principle, as a statement of the impossibility, by any internal physical measurements whatever, to detect uniform motion. This expression of the Principle remains however valid for mechanical experiments in both Galilean and Special Relativity, due to the covariance, in both theories, of Newton's Second Law.

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# 1 Introduction

The Hafele-Keating experiment (HKE) performed in October 1971 [1, 2], compared time intervals recorded by four caesium-beam atomic clocks, carried around the world in commercial aircraft, to those recorded by precision reference clocks at the U.S. Naval Laboratory. In the analysis of the experiment, the detailed flight paths of the aircraft on the West-to-East (W-E) and East-to-West (E-W) flights were taken into account in the prediction of special relativistic (SR) and general relativistic (GR), or gravitational, effects. Denoting the time intervals recorded by the Earthbound (airborne) clocks by  $T'$  ( $T''$ ) the following predictions were obtained [1]

$$\Delta T'(W - E) \equiv T''(W - E) - T'(W - E) = 144 \pm 14\text{ns}, \quad (\text{GR})$$

$$= -184 \pm 18\text{ns}. \quad (\text{SR})$$

$$\Delta T'(E - W) \equiv T''(E - W) - T'(E - W) = 179 \pm 18\text{ns}, \quad (\text{GR})$$

$$= 96 \pm 10\text{ns}. \quad (\text{SR})$$

The physical origin of the GR effect is gravitational blue-shift of the airborne clock relative to one on the surface of the Earth due to its higher gravitational potential <sup>1</sup> and so is determined by the mean altitude of the aircraft. The SR contribution to  $\Delta T'$  arises from the time dilation (TD) effect. In calculating it, the concept of ‘coordinate time’ [3, 4], or the equivalent one of ‘base’ and ‘travelling’ frames [5, 6], to be explained below, is essential. Coordinate time is registered by a hypothetical clock in a non-rotating frame<sup>2</sup> comoving with the centre of the Earth. This is the Earth Centered Inertial (ECI) frame which is also used in the analysis of SR effects in the GPS system [7]. The experimental results of the HKE were found to be in good agreement with the above predictions [2]:

$$\Delta T'(W - E) = -59 \pm 10\text{ns}, \quad (\text{Experiment})$$

$$= -40 \pm 23\text{ns}. \quad ((\text{GR}) + (\text{SR}))$$

$$\Delta T'(E - W) = 273 \pm 7\text{ns}, \quad (\text{Experiment})$$

$$= 275 \pm 21\text{ns}. \quad ((\text{GR}) + (\text{SR}))$$

Other experiments in which SR and GR effects near the Earth were measured and demonstrated to be in good agreement with theoretical predictions are described in Refs. [8, 9, 10]. In the experiment of Vessot *et al* [8], the SR and GR corrections to the rate of a hydrogen-maser clock flown in a rocket on a ballistic trajectory with a maximum altitude of  $10^4\text{km}$  were of comparable magnitude and opposite sign so that exact cancellation of the effects was observed during both the ascent and descent of the rocket. In the work described in Ref. [9] the altitude-dependence of the GR effect was tested by flying an aircraft carrying an array of caesium-beam atomic clocks on a closed path with a racetrack-like configuration over Chesapeake Bay in the State of Maryland USA. The same array of clocks was also flown from Washington DC to Thule, Greenland, and back,

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<sup>1</sup>The gravitational potential due to the Earth is negative and, assuming spherical symmetry, on and above the surface of the Earth is proportional to  $1/r$  where  $r$  is the distance from the center of the Earth.

<sup>2</sup>In the approximation that the rotation of the Earth around the centroid of the Solar System is neglected, this frame is an inertial one.

to check the predicted almost-exact cancellation [11, 12] of SR and GR effects on the rate of clocks placed on the Earth’s geoid at different latitudes. In the Spacelab experiment NAVEX [10] a caesium-beam atomic clock carried by a space shuttle in low-Earth orbit was compared to similar ground-based clocks, and the sum of the SR and GR predictions for the relative rate of the clocks was verified. As described in Ref. [7] TD and the general relativistic blue-shift give very large effects on the rates of the satellite-borne clocks of the GPS system, which are compensated for by adjusting the rates of the clocks on the surface of the Earth before they are sent into orbit. The HKE is however unique in its first demonstration of asymmetric time dilation effects for clocks on W–E and E–W paths. This is because, as pointed out by Hafele [4], satellite-borne clocks have speeds much greater than Earth-bound ones, so that the time dilation asymmetry in Eq. (2.8) below is imperceptible for them. Only in the HKE are the speeds, in the ECI frame, of the moving and Earth-bound clocks of similar magnitude. The asymmetrical time dilation effect has been called in the literature (as discussed in Appendix C below, inappropriately) a ‘Sagnac Effect’. The demonstration, in the HKE, that *the relative rate of two clocks does not depend only on their relative velocity* is of a crucial importance for a correct understanding of temporal predictions in SR [3, 4, 5, 6].

Asymmetrical time dilation is pure SR effect that can be seen to be an immediate consequence of the Minkowski metric equation of SR for space-time intervals:

$$(ds)^2 = c^2(\Delta\tau)^2 = c^2(\Delta t)^2 - (\Delta\vec{x})^2. \quad (1.1)$$

Suppose that two clocks C’, C’’ with proper frames S’, S’’ registering times  $t'$ ,  $t''$  have the world lines  $\Delta\vec{x}(C') = \vec{v}(C')\Delta t$ ,  $\Delta\vec{x}(C'') = \vec{v}(C'')\Delta t$ , in the frame S. It then follows from (1.1) that:

$$c^2(\Delta t')^2 = (c^2 - v(C')^2)(\Delta t)^2, \quad c^2(\Delta t'')^2 = (c^2 - v(C'')^2)(\Delta t)^2 \quad (1.2)$$

where  $v \equiv |\vec{v}|$ , so that

$$\Delta t = \gamma(C')\Delta t' = \gamma(C'')\Delta t'' \quad (1.3)$$

where

$$\gamma(C') \equiv \frac{1}{\sqrt{1 - \frac{v(C')^2}{c^2}}}, \quad \gamma(C'') \equiv \frac{1}{\sqrt{1 - \frac{v(C'')^2}{c^2}}}$$

It is immediately obvious, on inspection of the last member of (1.3), that the ratio of the proper time intervals  $\Delta t'$ ,  $\Delta t''$ , registered by the clocks C’ and C’’, depends not on the relative velocity  $|\vec{v}(C'') - \vec{v}(C')|$  of the clocks but on the separate and independent values of  $v(C')$  and  $v(C'')$ . These velocities of the clocks, are specified in the frame S, the ‘base frame’ in which the initial conditions of the problem are defined [5, 6].

In the present paper, SR effects in a simplified version of the HKE will be considered in which the aircraft moves with constant speed along Great Circles, the normal to which makes a variable angle  $\psi$  with the axis of rotation of the Earth. GR effects are neglected. Since, however, the important quantities in the analyses considered are double differences such as<sup>3</sup>:

$$\Delta T''(+ -) \equiv \Delta T'(W - E) - \Delta T'(E - W). \quad (1.4)$$

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<sup>3</sup>Note that, since  $T'$  cancels on the right side of (1.4), in the simplified version of the HKE considered here, the left side depends only on  $T''(W - E)$  and  $T''(E - W)$ . However, as discussed below, precise experimental determinaton of  $\Delta T''(+ -)$  requires separate measurements of  $\Delta T'(W - E)$  and  $\Delta T'(E - W)$ .

GR contributions will cancel provided the average altitude of the aircraft is constant during its flights. There remains a GR *correction* to the value of  $\Delta T''(+ -)$  which is tiny in comparison with the experimental uncertainties of the time interval measurements performed in the HKE. This correction is evaluated in the full GR analysis of Appendix B, and is given in Section 2 below. It applies also to the experiments described in Sections 3 and 5, but not to the free-space experiment of Section 4 where the SR formula (2.7) below gives an exact (to all orders in  $\beta$ ) prediction.

In the following section it is shown how observations of  $\Delta T''(+ -)$  for Great Circle flights, as a function of  $\psi$ , enable the angular momentum vector,  $\vec{\Omega}$ , of the Earth to be determined from purely internal measurements of differences of time intervals recorded by airborne and Earthbound clocks. In Section 3 a method is described to determine the speed and direction of motion of an arbitrary point on the Earth's surface by local measurements, using a variable which is a generalisation of  $\Delta T''(+ -)$ , by comparing airborne clocks with several synchronised clocks at rest on the surface of the Earth. Section 4 contains a straightforward generalisation to three spatial dimensions of the analysis of the previous section, in order to measure internally the vectorial velocity of an arbitrary travelling inertial frame in free space. Section 5 shows how both the velocity of a ship relative to the surface of the Earth (as considered by Galileo) as well as the local velocity of the surface of the Earth may, in principle, be derived by observing the difference between time intervals recorded by a moving clock on the ship and an array of synchronised fixed clocks at different positions on the ship —again by purely internal measurements. The final section contains a summary and conclusions.

## 2 Internal measurement of the angular velocity $\vec{\Omega}$ of the Earth using an aircraft and two clocks

In the following discussion, only SR effects are considered, and the Earth is assumed to be exactly spherical and of radius  $R$ . For the case of equatorial circumnavigation, the initial conditions of the experiment are completely specified by two parameters,  $v_E \equiv v(C') \equiv \Omega R$ , which is the speed of the clock,  $C'$  (with proper frame  $S'$ ), at rest on the surface of the Earth, in a non-rotating inertial frame,  $S$ , comoving with the centre of the Earth, and  $v'_A \equiv v'(C'')$ , which is the speed of the airborne clock,  $C''$ , (with proper frame  $S''$ ) in the frame  $S'$ . For the analysis of the experiment it is necessary to introduce the coordinate time recorded by a hypothetical clock  $C$  at rest in  $S$ . In the nomenclature for space-time experiments recently introduced by the present author [5, 6] the frame  $S$  is termed the ‘base frame’ of the experiment while  $S'$  and  $S''$  constitute different ‘travelling frames’. The base frame is defined as the one in which the clock is at rest in a time dilation (TD) experiment. If, in such an experiment, a moving clock is seen to run slow (fast) then the corresponding observer is in the base (travelling) frame of the experiment. In the experiments to be discussed below, the time <sup>4</sup>,  $t$ , registered by the clock  $C$ , remains

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<sup>4</sup>The word ‘time’, used without qualification, means a number, registered by a clock at some instant,

hypothetical since no observations of events in the external inertial frame  $S$  are performed. It is shown, however, that purely internal observations of the times  $t'$  and  $t''$  recorded by  $C'$  and  $C''$  respectively are sufficient to establish the travelling nature of the latter frames and to determine the base frame velocity  $v_E$  and hence  $\Omega$ . It will be seen that, in the lowest order approximation, knowledge of  $v'_A$  is needed only to determine the value of  $R$ .

The frames  $S'$  and  $S''$  are not inertial frames—they are subjected to uniform transverse accelerations. It was demonstrated, by observation of time-dilated decay lifetimes of ultra-relativistic muons with  $\gamma = 29$  ( $\gamma \equiv 1/\sqrt{1 - (\vec{v}/c)^2}$ ) in near-circular orbits in a storage ring at CERN [13], that the TD effect is the same as for muons in an inertial frame with velocity equal to the magnitude,  $|\vec{v}|$ , of their average velocity  $|\vec{v}|$ , in the presence of a transverse acceleration of  $10^{19}g$ . It is therefore assumed, in the following, that  $S'$  and  $S''$  may be considered to be inertial frames for calculations of the TD effect. A similar assumption is made in the calculation of SR corrections due to TD effects for the satellite-borne clocks of the GPS system [7].

The experiments to be considered in this section are generalisations of the HKE in which the aircraft moves with constant speed along an arbitrary Great Circle and the Earthbound clock  $C'$  is situated at an arbitrary (and initially unknown) position on the Earth's surface. A solution is sought to the following problem, supposing that the surface of the Earth may be considered to be perfectly spherical:

Determine the size, axis of rotation, and speed of rotation of the Earth from purely internal observations of two clocks, one at rest on the surface of the Earth and the other in an aircraft that moves at constant speed  $v'_A$  relative to the surface of the Earth, along a Great Circle.

It is also assumed that the clock  $C'$  may be moved in an arbitrary manner (transported, for example, by the aircraft) over the surface of the Earth. Circumstances under which the solution of the problem would be of practical importance would be, for example, if the whole Earth were blanketed in an opaque cloud following an asteroid collision, or a massive volcanic eruption, so that the Sun and fixed stars were no longer visible from the Earth, and it was required to find the the axis of rotation of the Earth, i.e. the positions of the North and South Poles.

The geometry, in the case that the angle between the Earth's axis of rotation ( $z'$ -axis) and the normal to the plane of the Great Circle followed by the aircraft ( $z$ -axis) is  $\psi$ , is shown in perspective view in Fig. 1. The common  $x, x'$  axis is perpendicular to the plane spanned by the  $z$  and  $z'$  axes, while the  $y$  and  $y'$  axes complete right-handed Cartesian coordinate systems. Projections into the  $x, y$  and  $y, z$  planes are shown in Fig. 2a and 2b respectively. The position of the aircraft, along the Great Circle, is specified by the azimuthal angle  $\phi$ . The angle,  $\theta$ , between the direction of flight of the aircraft and the tangent vector, in the W–E direction, to a circle of fixed latitude, is given by the relation, derived in Appendix A

$$\cos \theta = \frac{\cos \psi}{[1 - \sin^2 \psi \sin^2 \phi]^{\frac{1}{2}}} \quad (2.1)$$

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also commonly called the 'epoch' of the clock.

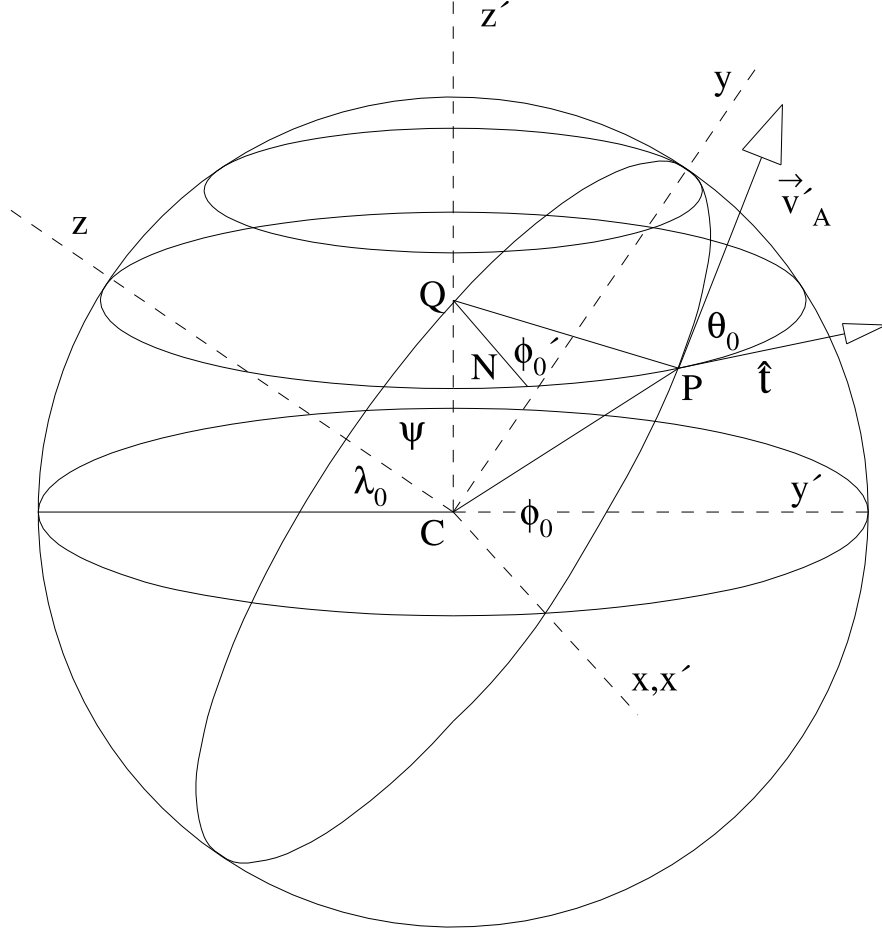


Figure 1: *Perspective view of the Great Circle path followed by the aircraft with velocity  $\vec{v}'_A$  relative to the fixed point  $P$  on the surface of the Earth. The normal to the Great Circle (the  $z$ -axis) is inclined at an angle  $\psi$  relative to the polar  $z'$ -axis. The unit vector  $\hat{t}$  is the tangent, in the  $W$ - $E$  direction, at  $P$ , to the circle of fixed latitude  $\lambda_0$ . The  $x, x'$ -axis is normal to the plane spanned by the  $z$ - and  $z'$ -axes and its projection into the circle of fixed latitude is  $QN$ . The angle between  $\vec{v}'_A$  and  $\hat{t}$  is  $\theta_0$ , and  $\phi_0, \phi'_0$  are azimuthal coordinates of  $P$  along the Great Circle and the circle of fixed latitude, respectively. The  $x, x'$  and  $y'$  axes lie in the Earth's equatorial plane, the  $x, x'$  and  $y$  axes in the plane of the Great Circle.*

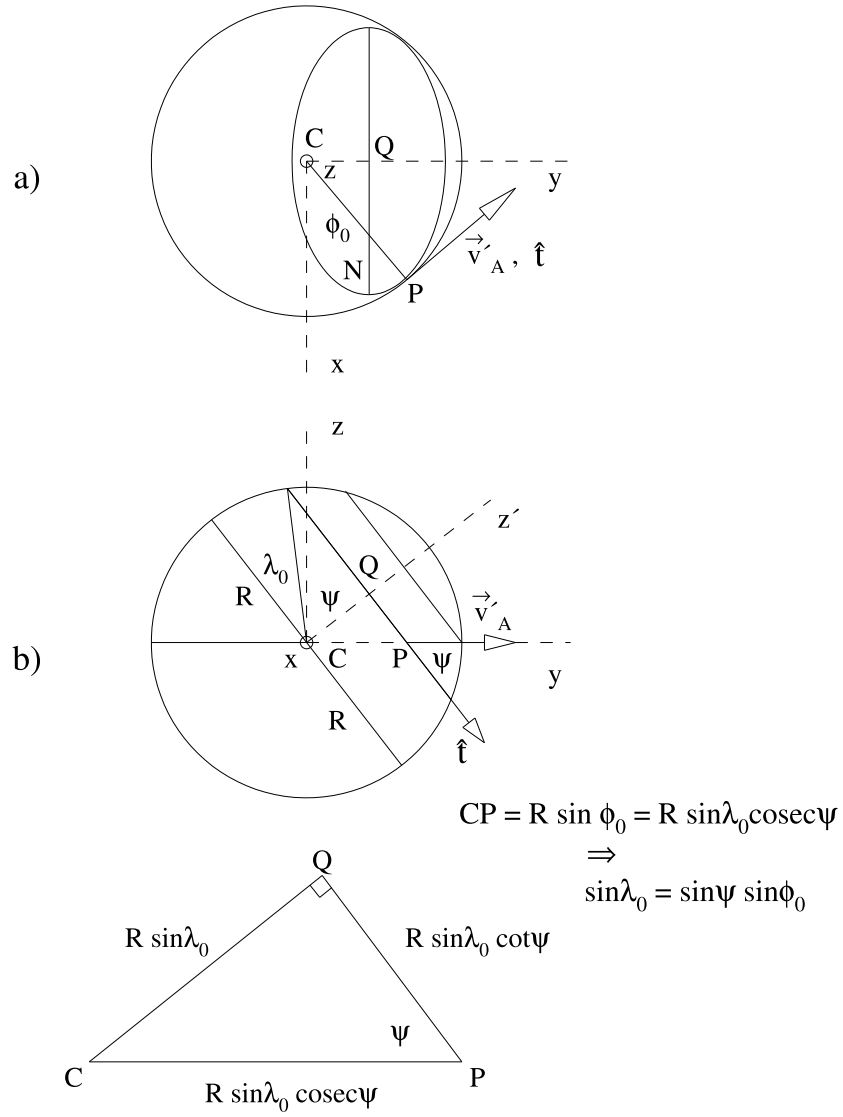


Figure 2: a)  $x, y$  and b)  $y, z$  projections of the configuration of Fig.1.  $R$  is the radius of the Great Circle in a) and of the Earth in b).

while the geometry of Fig. 2b gives the latitude angle,  $\lambda_0$ , at P, in terms of  $\psi$  and  $\phi_0$  as

$$\sin \lambda_0 = \sin \psi \sin \phi_0. \quad (2.2)$$

The differential TD relations for the clocks  $C'$  and  $C''$  at an arbitrary point on the great circle with latitude angle  $\lambda$  are (c.f. Eq. (1.3)):

$$dt = \gamma(C', \lambda) dt' = \gamma(C'', \lambda, \theta) dt'' \quad (2.3)$$

where

$$\gamma(C', \lambda) \equiv \frac{1}{\sqrt{1 - \beta(C', \lambda)^2}}, \quad \beta(C', \lambda) \equiv \frac{v_E(\lambda)}{c} \equiv \frac{\Omega R \cos \lambda}{c}, \quad v_E \equiv v_E(\lambda = 0) \quad (2.4)$$

and  $c$  is the speed of light in free space. Since the TD factor,  $\gamma$ , transforms as the temporal component of the dimensionless velocity four-vector:  $(\gamma, \gamma\vec{\beta})$ , it follows that:

$$\gamma(C'', \lambda, \theta) = \gamma(C', \lambda) [\gamma'(C'') + \beta(C', \lambda) \beta'(C'') \gamma'(C'') \cos \theta] \quad (2.5)$$

where

$$\gamma'(C'') \equiv \frac{1}{\sqrt{1 - \beta'(C'')^2}}, \quad \beta'(C'') \equiv \frac{v'_A}{c}. \quad (2.6)$$

Combining (2.3) and (2.5) gives:

$$dt'' = \frac{\gamma(C', \lambda)}{\gamma(C'', \lambda, \theta)} dt' = \frac{dt'}{\gamma'(C'') [1 + \beta(C', \lambda) \beta'(C'') \cos \theta]}. \quad (2.7)$$

Retaining only  $O(\beta^2)$  terms on the right side of (2.7) and making use of (2.1) and the generalisation of (2.2), for an arbitrary point on the Great Circle, gives:

$$dt'' = \left[ 1 - \frac{\beta'(C'')^2}{2} \mp \beta(C') \beta'(C'') \cos \psi \left( \frac{1 - \sin^2 \psi \cos^2 \phi}{1 - \sin^2 \psi \sin^2 \phi} \right)^{\frac{1}{2}} \right] dt' \quad (2.8)$$

where  $\beta(C') \equiv \beta(C', \lambda = 0)$  and the  $- (+)$  signs correspond to  $\cos \theta > (<) 0$ . Since the aircraft moves with uniform speed around the Great Circle,  $dt' = (R/v'_A) d\phi$ . Also (2.1) and (2.2) may be used to derive the following relation (see Appendix A) between  $\psi$  and the values,  $\theta_0$  and  $\lambda_0$ , of  $\theta$  and  $\lambda$ , at the point P:

$$\cos \psi = \cos \theta_0 \cos \lambda_0. \quad (2.9)$$

The formula (2.8) which is the basis for all the calculations presented in the present and subsequent sections of the present paper was first derived for the special case:  $\cos \theta > 0$  and  $\psi = 0$  by Hafele [3, 4]. Since it gives the ratio of infinitesimal proper time intervals, it does not depend on the shape of the world lines followed, in the frame S, by the clocks  $C'$  and  $C''$ . In particular it is equally valid for the circular world lines considered in the present section and the straight ones considered in Sections 3, 4 and 5 below, where the proper frames of  $C'$  and  $C''$  are inertial ones. Hafele's GR calculation, yielding (2.8), in the limit of a vanishing or constant gravitational field, is recalled in Appendix B below. It was also derived by Hafele as a pure SR calculation from the Minkowski metric (as in

the derivation of Eq. (1.3) in the Introduction of the present paper) in the Appendix of Ref. [4].

Labelling the Great Circle followed by the aircraft as ‘+’ if  $\cos \theta_0 > 0$  (i.e. one with W–E motion) and ‘–’ if  $\cos \theta_0 < 0$  (i.e. one with E–W motion) the round trip time  $T''$  as recorded by  $C''$  is given by (2.8) as:

$$\begin{aligned} T''(\theta_0, \lambda_0, \pm) &= \frac{R}{v'_A} \int_{\phi_0}^{2\pi+\phi_0} \left[ 1 - \frac{\beta'(C'')^2}{2} \mp \beta(C')\beta'(C'') \left( \frac{1 - \sin^2 \psi \cos^2 \phi}{1 - \sin^2 \psi \sin^2 \phi} \right)^{\frac{1}{2}} \cos \psi \right] d\phi \\ &= \frac{2\pi R}{v'_A} \left[ 1 - \frac{\beta'(C'')^2}{2} \mp \frac{\beta(C')\beta'(C'')I(\psi)}{2\pi} \right] \end{aligned} \quad (2.10)$$

where

$$I(\psi) \equiv \int_0^{2\pi} \left( \frac{1 - \sin^2 \psi \cos^2 \phi}{1 - \sin^2 \psi \sin^2 \phi} \right)^{\frac{1}{2}} \cos \psi d\phi \quad (2.11)$$

and  $\psi$  is given, in terms of  $\theta_0$  and  $\lambda_0$ , by (2.9) with  $0 < \theta_0 < \pi/2$ . Note that the value of the  $\phi$  integral in (2.10) is independent of  $\phi_0$ . The duration,  $T'$ , of a Great Circle flight, as recorded by  $C'$ , is independent of the value of  $\psi$ :

$$T' = \frac{2\pi R}{v'_A}. \quad (2.12)$$

The difference of the time intervals recorded by  $C''$  and  $C'$ ,  $\Delta T'$  is then, from (2.10) and (2.12):

$$\Delta T'(\theta_0, \lambda_0, \pm) \equiv T''(\theta_0, \lambda_0, \pm) - T' = T' \left[ -\frac{\beta'(C'')^2}{2} \mp \frac{\beta(C')\beta'(C'')I(\psi)}{2\pi} \right]. \quad (2.13)$$

It follows from (2.13) that:

$$\begin{aligned} \Delta T''(\theta_0, \lambda_0, +- ) &\equiv \Delta T'(\theta_0, \lambda_0, +) - \Delta T'(\theta_0, \lambda_0, -) \\ &= -\frac{T'\beta(C')\beta'(C'')I(\psi)}{\pi} = -\frac{2Rv_E I(\psi)}{c^2}. \end{aligned} \quad (2.14)$$

Notice that, in this approximation where terms of  $O(\beta^4)$  and higher are neglected, the value of  $\Delta T''(\theta_0, \lambda_0, +-)$  depends only on  $c$ , the radius of the Earth and the values of  $\psi$  and  $v_E$ , *being independent of the speed of the aircraft,  $v'_A$  in the frame  $S'$ .*

Curves of  $\Delta T'(\theta_0, \lambda_0, +)$ ,  $\Delta T'(\theta_0, \lambda_0, -)$  and  $\Delta T''(\theta_0, \lambda_0, +-)$  as a function of  $\theta_0$  for various values of  $\lambda_0$  are shown in Figs. 3, 4 and 5 respectively. The parameter values used:  $T' = 27\text{h}$ ,  $v_E = 470\text{m/s}$  and  $v'_A = 300\text{m/s}$  are similar to those of the actual HKE [2] and the integral  $I(\psi)$  is evaluated numerically. As just mentioned, the curves in Fig. 5 are independent of the value of  $v'_A$ . The radius of the Earth is determined from (2.12) by the observed value of  $T'$ , from the known value of  $v'_A$ . The direction of the line of fixed longitude at P is determined by the orientation of the Great Circle for which  $\Delta T''(\theta_0, \lambda_0, +-)$  vanishes ( $\theta_0 = 90^\circ$  in Fig. 5). The line of fixed latitude at P is perpendicular to this direction. The direction of rotation of the Earth is determined by the sign of  $\Delta T''(\theta_0 = 0, \lambda_0, +-)$ . If it is positive (negative) the aircraft moves in

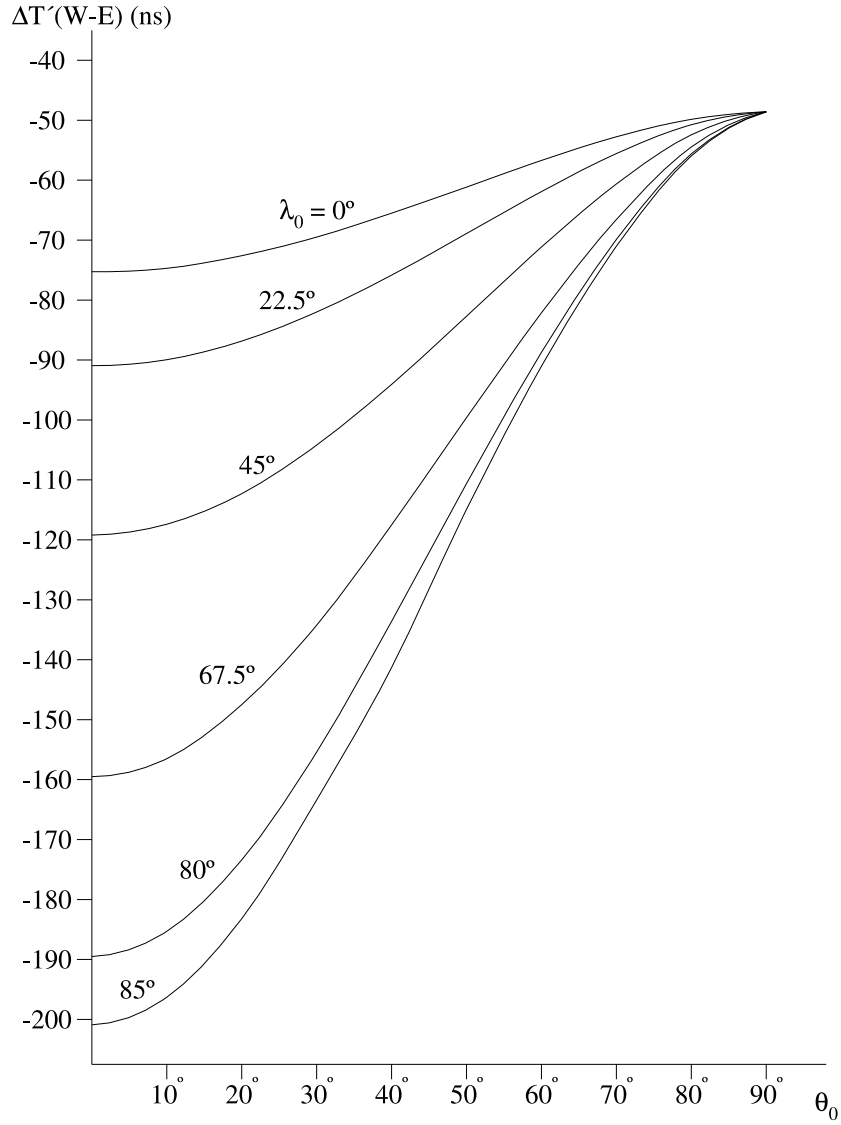


Figure 3:  $\Delta T'(W - E)$  (in ns) as a function of  $\theta_0$  for fixed values of the latitude angle  $\lambda_0$  of the point  $P$  in Fig. 1 at which the Great Circle flight starts.  $T' = 27h$ ,  $v_E = 470m/s$  and  $v'_A = 300m/s$ .

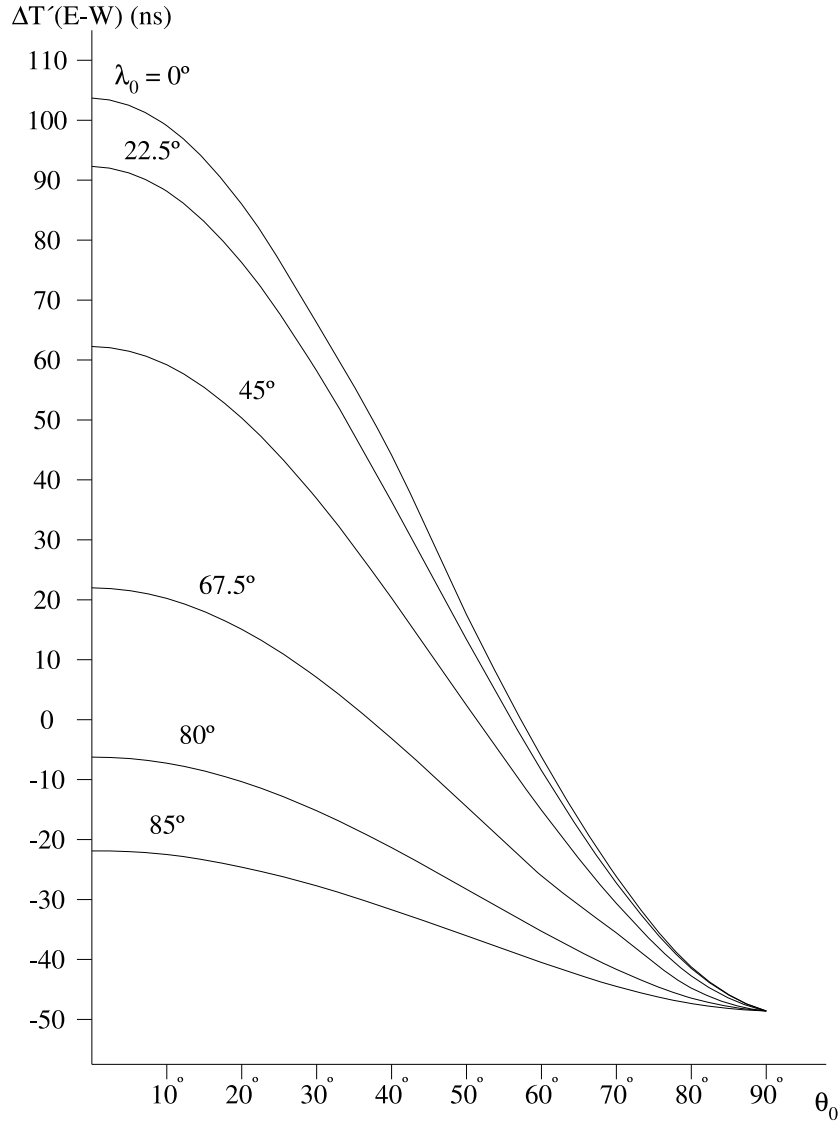


Figure 4:  $\Delta T'(E - W)$  (in ns) as a function of  $\theta_0$  for fixed values of the latitude angle  $\lambda_0$  of the point  $P$  in Fig. 1 at which the Great Circle flight starts.  $T' = 27h$ ,  $v_E = 470m/s$  and  $v'_A = 300m/s$ .

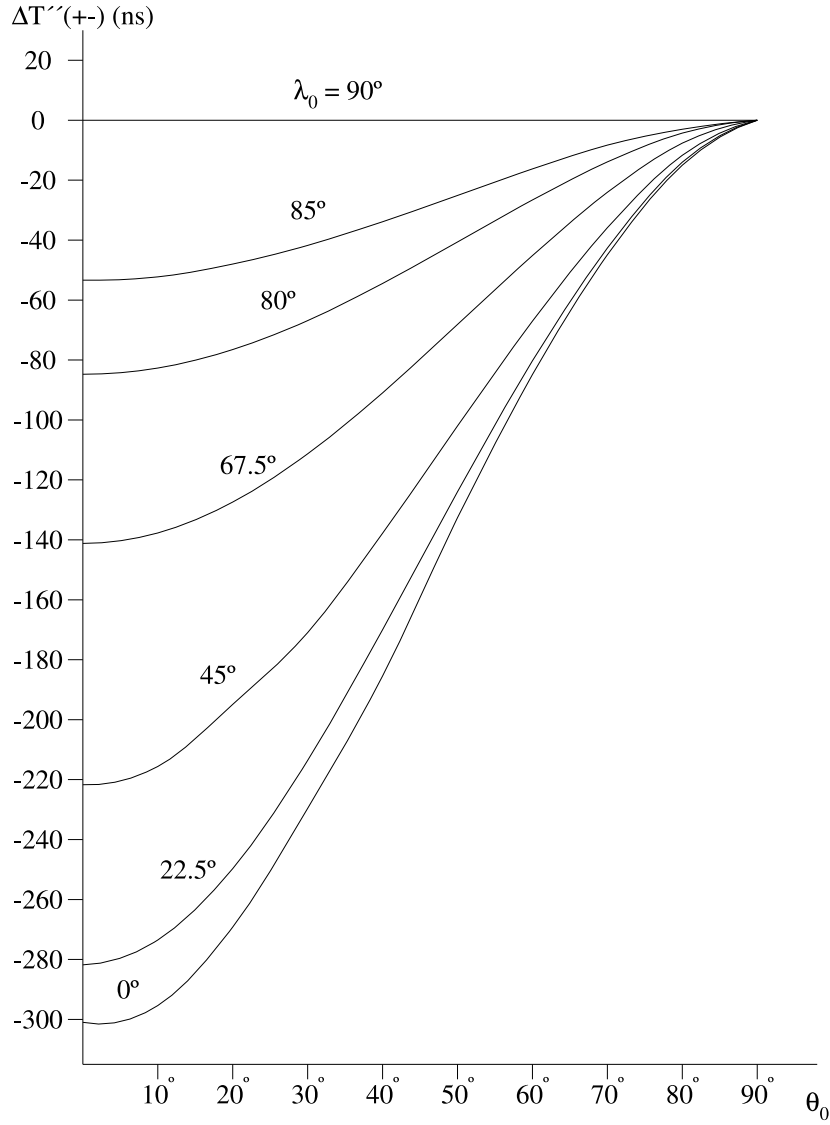


Figure 5:  $\Delta T''(+ -) \equiv \Delta T'(W - E) - \Delta T'(E - W)$  (in ns) as a function of  $\theta_0$  for fixed values of the latitude angle  $\lambda_0$  of the point  $P$  in Fig. 1 at which the Great Circle flight starts. Note that the curves are independent of the value of  $v'_A$ .  $T' = 27h$ ,  $v_E = 470m/s$ .

the E–W (W–E) direction. To now determine the value of  $v_E(\lambda = 0)$ , and hence  $|\vec{\Omega}| = v_E(\lambda = 0)/R$ , the clock  $C'$  is transported along the line of longitude in the direction of increasing  $|\Delta T''(\theta_0 = 0, \lambda_0, +-)|$  until the maximum value of this quantity is obtained. The corresponding Great Circle is the Equator ( $\theta_0 = \lambda_0 = \psi = 0$ ). Since then, from (2.11),  $I(\psi = 0) = 2\pi$ , it follows from (2.14) that:

$$|\vec{\Omega}| = \frac{v_E}{R} = c^2 \frac{|\Delta T''(\theta_0 = 0, \lambda_0 = 0, +-)|}{4\pi R^2}. \quad (2.15)$$

The radius of the Earth and the position of the Equator, as well as the positions of the North and South Poles (i.e the direction of the axis of rotation of the Earth), are now known. Since the direction of rotation along lines of constant latitude, as well as the value of  $|\vec{\Omega}|$ , are also known,  $\vec{\Omega}$  is determined and the problem posed above is solved.

Inclusion of GR effects results in a correction factor  $1 + 2\phi_E/c^2$  on the right side of (2.15) where  $\phi_E = -GM_E/R = -6.25 \times 10^{11}(\text{m/s})^2$ , so that the correction factor has the value  $1 - 1.38 \times 10^{-5}$ , to be compared with a relative experimental error of about 15% on the measurement of  $\Delta T''(+)$  in the HKE experiment.

It has become customary in the literature [14, 15] to call the formulae (2.15) which embodies the asymmetric time dilation effect observed in the HKE experiment, a ‘Sagnac Effect’ due to the accidental mathematical identity of the equation with that giving the difference of shifts of arrival times of light signals, travelling around the Earth along its Equator, in the W–E or E–W directions, due to the Earth’s easterly rotation. This is a purely classical  $O(\beta)$  effect, a consequence of differing relative velocities of the light signal (which moves with speed  $c$  in the ECI frame) and the receiver on the surface of the Earth. In contrast Eq. (2.15) arises from the purely relativistic ( $O(\geq \beta^2)$ ) TD effect. The Sagnac Effect formula is derived and discussed, in comparison with the TD formula (2.15), in Appendix C. The incorrect classification of the classical Sagnac effect as a relativistic one has also recently been pointed out by Gezari [16].

Although originally observed in a rotating interferometer [17], the Sagnac Effect also occurs for a receiver in uniform motion relative to the light signal [18]. More recent studies show that the effect can be used to construct an accurate detector for uniform motion [19]. Thus the method to detect such motion, based on the asymmetric time dilation effect described in the present paper is not unique. The asymmetric time dilation effect is a purely relativistic one whereas the Sagnac Effect is classical at lowest order, arising from different, classically defined, relative velocities between a light signal and its receiver. Both types of experiments demonstrate that internal detection of uniform translational motion is possible, contrary to the version of the Special Relativity Principle that states the impossibility to detect such motion.

### 3 Internal measurement of the speed of motion of an arbitrary point on the Earth’s surface using a moveable clock

Over limited regions of the Earth’s surface, the latitude-dependent velocity  $v_E(\lambda)$

may be considered constant. For example, for displacements of  $\pm 100\text{km}$  along a line of longitude, near  $\lambda = 45^\circ$  the value of  $v_E(\lambda)$  changes by only  $\pm 1.6\%$ . The average value of  $v_E(\lambda)$  in such a restricted region can be measured locally by taking advantage of the same frame-dependent time dilation effects as manifested in the HKE.

An experimental setup for such a measurement is shown schematically in Figs. 6 and 7. The principle of the measurement is illustrated in Fig. 6. Clocks  $C'(O)$ ,  $C'(x)$  and  $C'(y)$  are placed, respectively, at the origin and at equal distances,  $D$ , along the  $x$ - and  $y$ -axes of an arbitrarily-oriented Cartesian coordinate system on the surface of the Earth. The above three clocks are synchronised, and a fourth clock,  $C''$ , moves along the  $x$ - or  $y$ -axes at constant speed  $v'$ . Some different methods of synchronising spatially-separated clocks are described in Ref. [20]. A convenient method for the experiment described here, as well as that of Section 5, is to use a signal cable<sup>5</sup> with a known delay time  $t_D$ . The distant to-be-synchronised clock is stopped and set to show the time  $t_D$ . The local clock is started, with initial setting zero, at the instant that the synchronisation signal is sent along the cable. The distant clock, started on reception of the signal, is then synchronised with the local clock. It is important to remark that the ‘slow clock transport method’ [21] cannot be used to synchronise separated clocks on the surface of the Earth unless they lie on the same line of longitude ( $\theta = 90^\circ$  in Eq. (2.7)). The time intervals recorded by  $C''$  as it moves from the position of clock  $C'(O)$  to that of  $C'(x)$ ,  $T''(x, +)$ , or *vice versa*,  $T''(x, -)$ , are compared with those recorded by  $C'(O)$  and  $C'(x)$ . Similar comparisons are made for the intervals  $T''(y, +)$  and  $T''(y, -)$  which are similarly defined for motion along the  $y$ -axis. The geometry of Fig. 6 and (2.7) gives, retaining only  $O(\beta^2)$  terms on the right side of (2.7):

$$\Delta T'(x, +) \equiv T''(x, +) - T' = \frac{D}{v'} \left[ -\frac{1}{2} \left( \frac{v'}{c} \right)^2 - \frac{v'v_E(\lambda)}{c^2} \cos \theta \right], \quad (3.1)$$

$$\Delta T'(x, -) \equiv T''(x, -) - T' = \frac{D}{v'} \left[ -\frac{1}{2} \left( \frac{v'}{c} \right)^2 + \frac{v'v_E(\lambda)}{c^2} \cos \theta \right], \quad (3.2)$$

$$\Delta T'(y, +) \equiv T''(y, +) - T' = \frac{D}{v'} \left[ -\frac{1}{2} \left( \frac{v'}{c} \right)^2 + \frac{v'v_E(\lambda)}{c^2} \sin \theta \right], \quad (3.3)$$

$$\Delta T'(y, -) \equiv T''(y, -) - T' = \frac{D}{v'} \left[ -\frac{1}{2} \left( \frac{v'}{c} \right)^2 - \frac{v'v_E(\lambda)}{c^2} \sin \theta \right]. \quad (3.4)$$

where  $T' = D/v'$  is the time interval recorded by  $C'(O)$ ,  $C'(x)$  and  $C'(y)$  during the passages of  $C''$ . Eq. (3.1) shows that  $T''(x, +) - T' < 0$  for  $\cos \theta > 0$ , so that, as shown in Fig. 6a, for motion in the positive  $x$ -direction,  $C''$  is seen to run slow relative to clocks at rest on the surface of the Earth. In the case that  $v_E(\lambda) \cos \theta > v'/2$ , (3.2) gives  $T''(x, -) - T' > 0$  and, as shown in Fig. 6b, for motion in the negative  $x$ -direction,  $C''$  runs *fast* relative to such clocks. Fig. 6 shows clearly that the observed TD effect in the frame  $S'$  does not depend only on the relative velocity of the stationary and moving clocks in the experiment, as might naively be expected if ‘everything is relative’. Indeed, in Fig. 6b there is a ‘time contraction’, not a time dilation effect, as the moving clock actually

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<sup>5</sup>This should be a conventional cable with a metallic conductor in which the signal is carried by conduction electrons. The light signal propagation time in a fiber-optics cable depends on the motion of the cable due to the Sagnac effect [18, 19].

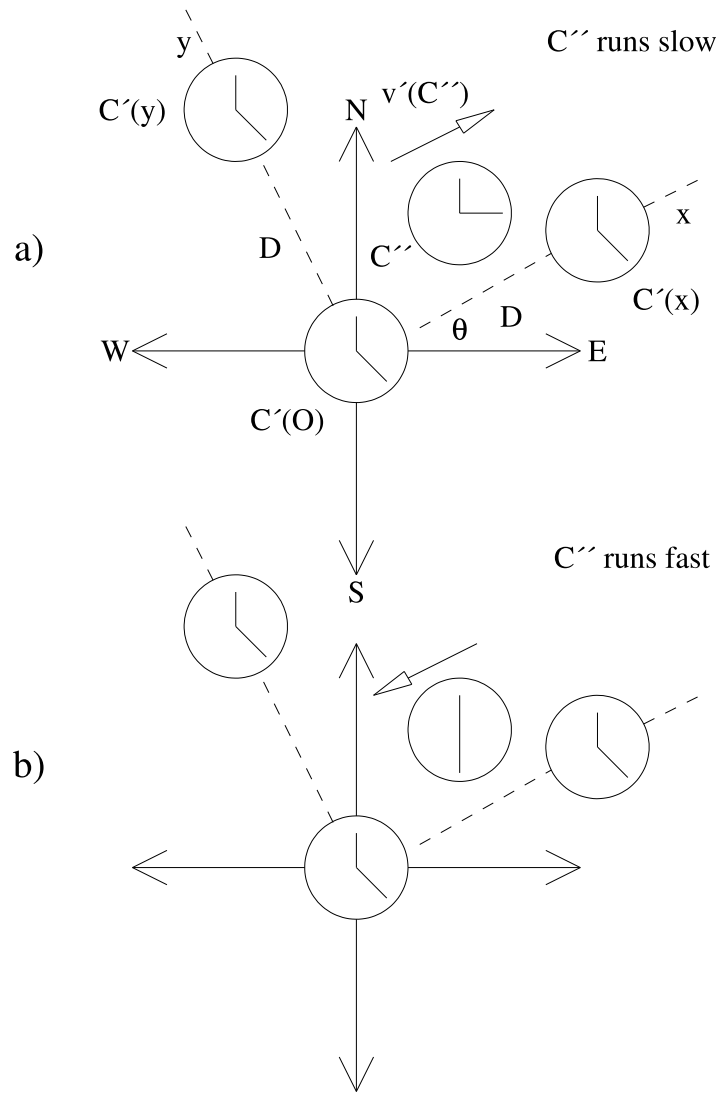


Figure 6: Arrangement of clocks for a local measurement of the velocity, in the frame  $S$ , of a point on the Earth's surface.  $C'(O)$ ,  $C'(x)$  and  $C'(y)$  are synchronised clocks, at fixed locations, between which the clock  $C''$  is moved at constant speed. In a) where  $C''$  moves along the positive  $x$ -axis it is seen by observers at rest on the Earth's surface to run slow relative to the clocks  $C'$ . In b) where  $C''$  moves in the negative  $x$ -direction, at the same speed, it is seen to run fast. Synchronised clocks (not shown) each recording the time shown by  $C'$  are located at the marker positions:  $O$ ,  $X$  and  $Y$ . See text for discussion.

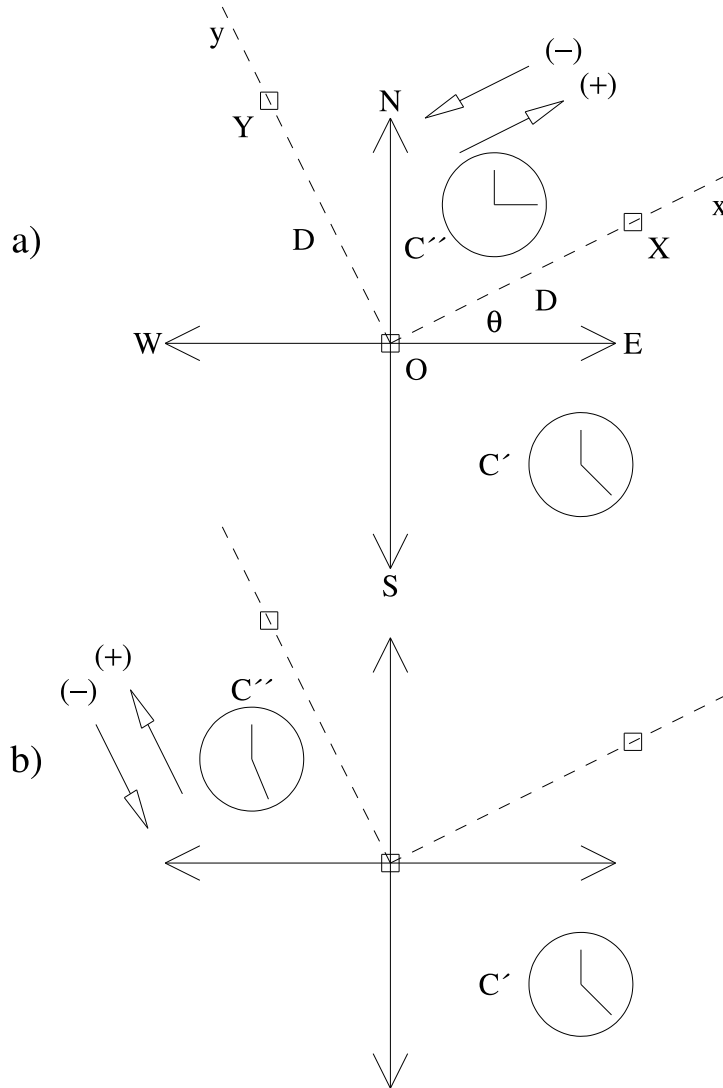


Figure 7: a) Measurement of  $\Delta T''(x, +-)$  by moving  $C''$  along the  $x$ -axis. a) Measurement of  $\Delta T''(y, +-)$  by moving  $C''$  along the  $y$ -axis. Synchronised clocks (not shown) are located at the marker positions:  $O, X, Y$ . See text for discussion.

runs faster than the stationary ones. This demonstrates the fundamental importance of the concepts of ‘base’ and ‘travelling’ frames [5, 6], or the equivalent one of ‘coordinate time’ [3, 4] (recorded by a clock at rest in the frame in which the velocity of a moving clock is specified) for the correct description of temporal effects in space-time experiments.

It follows from (3.1)–(3.4) that:

$$\Delta T'(x, +) - \Delta T'(x, -) \equiv \Delta T''(x, +- ) = -\frac{2Dv_E(\lambda)}{c^2} \cos \theta, \quad (3.5)$$

$$\Delta T'(y, +) - \Delta T'(y, -) \equiv \Delta T''(y, +- ) = \frac{2Dv_E(\lambda)}{c^2} \sin \theta. \quad (3.6)$$

Notice that although the time interval  $T'$  formally cancels in the definition of  $\Delta T''$ , it is crucial for the experiment that what can be observed with high precision —as done in the HKE— is only the *difference* between the time intervals recorded by  $C''$  and  $C'$ . This is given by comparing the readings of the two clocks, which if they run at the same constant rate can, in principle, be done at any time after  $C''$  is brought to rest in the frame  $S'$ . It is clearly practically impossible to measure the tiny difference between, say,  $T''(x, +)$  and  $T''(x, -)$  by attempting to measure these intervals simply by observing the epochs recorded by  $C''$  at the beginning and end of the transit. However, if this could be done, the clock  $C'$  would actually not be needed to determine  $\Delta T''(+ -)$ , and the experiment could be performed using the clock  $C''$  alone, thus avoiding completely the necessity to consider clock synchronisation..

The measurements of  $\Delta T''(x, +-)$  and  $\Delta T''(y, +-)$  now give the magnitude of the local velocity of the surface of the Earth, as well as the direction of the local line of fixed latitude:

$$v_E(\lambda) = \frac{c^2}{2D} [\Delta T''(x, +-)^2 + \Delta T''(y, +-)^2]^{\frac{1}{2}}, \quad (3.7)$$

$$\cos \theta = \frac{-\Delta T''(x, +-)}{[\Delta T''(x, +-)^2 + \Delta T''(y, +-)^2]^{\frac{1}{2}}}, \quad (3.8)$$

$$\sin \theta = \frac{\Delta T''(y, +-)}{[\Delta T''(x, +-)^2 + \Delta T''(y, +-)^2]^{\frac{1}{2}}}. \quad (3.9)$$

A scheme for an actual experiment is shown in Fig. 7. In some flat deserted region of the Earth, markers O, X and Y, each equipped with a synchronised clock, are set up at the origin and at equal distances,  $D$ , along the axes of a Cartesian coordinate system. The precision clock  $C''$  is placed in a helicopter and flown, at constant speed, from O to X then from X to O (Fig. 7a) and from O to Y then from Y to O (Fig. 7b). During the passages the four time interval differences  $T''(x, \pm) - T'$  and  $T''(y, \pm) - T'$  are measured. Performing the experiment at the Equator where  $v_E(\lambda = 0) = 470\text{m/s}$ , and setting  $D = 100\text{km}$ , gives  $\Delta T''(x, +- ) = 1.04\text{ns}$  for  $\theta = 0$ . If the speed of the helicopter is  $300\text{km/h}$  the time intervals  $T''$  must therefore be measured with a relative precision considerably better than  $\simeq 10^{-11}$  over the flight time  $T' = 20\text{m}$ . This is well within the capability of modern atomic clocks which can measure time intervals with relative precision of  $10^{-15}/\text{day}$  [22].

## 4 Internal measurement using moveable clocks of the velocity $\vec{v}$ of an arbitrary travelling frame in free space

The comoving frames  $S'$  and  $S''$  of the clocks  $C'$  and  $C''$  during the measurements described in the last section are not inertial frames, as they are subject to transverse acceleration. However as previously discussed and experimentally verified, the TD effects are the same as they would be in strictly inertial frames moving with the same total velocity relative to the frame  $S$ , in which coordinate time is defined. It is then evident that if the markers  $O$ ,  $X$  and  $Y$  shown in Fig. 7 were situated in free space and all moved with the same uniform velocity in the  $x$ - $y$  plane in  $S$ , the predictions of Eqs. (3.7)-(3.9) would be unchanged. Thus the velocity of an inertial travelling frame  $S'$ , relative to its base frame  $S$ , can, in general, be determined by purely internal measurements: observations of  $\Delta T''$ , in  $S'$ .

The measurement of the magnitude and direction, in a plane, of the velocity of  $S'$  relative to  $S$  given by Eqs. (3.7)-(3.9) is readily generalised to motion in three spatial dimensions. How this might be done is shown schematically in Fig. 8. The marker objects  $O, X, Y$  and  $Z$  are situated, respectively, at the origin and at equal distances,  $D$ , along arbitrarily oriented  $x$ -,  $y$ - and  $z$ -axes of a Cartesian coordinate system in free space. Each marker object is equipped with a synchronised clock:  $C'(O)$ ,  $C''(X)$ ,  $C'(Y)$  and  $C'(Z)$ . The clocks may conveniently be synchronised by the method described in the previous section, with light signals in free space replacing the cable-borne ones. The clocks at  $X$ ,  $Y$  and  $Z$  are stopped and set to  $t' = D/c$ . At time  $t' = 0$  the clock at  $O$  is started and light signals are sent towards the other clocks. The latter start on receiving the light signals. All four clocks are then synchronised. Since all the clocks are simultaneously accelerated in an identical manner they remain synchronised throughout the acceleration program. Notice that, as mentioned in the previous section, the clocks at  $O$ ,  $X$ ,  $Y$  and  $Z$  are only placed, for experimental convenience, to provide direct measurements of  $\Delta T'(+)$  and  $\Delta T'(-)$ . If the time intervals  $T''(+)$  and  $T''(-)$  could be directly observed only the single (moving) clock  $C''$  would suffice to perform the experiment. Initially, all the markers can be considered to be at rest in the frame  $S$ . They all undergo an identical and simultaneous acceleration program that holds constant their relative positions in their common comoving inertial frame. When each object has velocity  $\vec{v}$  in the frame  $S$  the acceleration is stopped and all the markers move thereafter with uniform velocity  $\vec{v}$ .

The magnitude and direction of  $\vec{v}$  is then internally determined, during the phase of uniform motion, by moving the clocks  $C''(x)$ ,  $C''(y)$  and  $C''(z)$ , between the marker objects, at constant speed along the coordinate axes as shown in Fig. 8, and observing the time interval difference  $\Delta T''$  between this clock and the synchronised fixed clocks at the marker positions. Of course, the same, physical, moving clock can be used to perform all the measurements. The geometry of Fig. 8 gives, as the three-dimensional generalisation of Eqs. (3.5) and (3.6):

$$\Delta T''(z, +- ) = -\frac{2Dv}{c^2} \cos \theta, \quad (4.1)$$

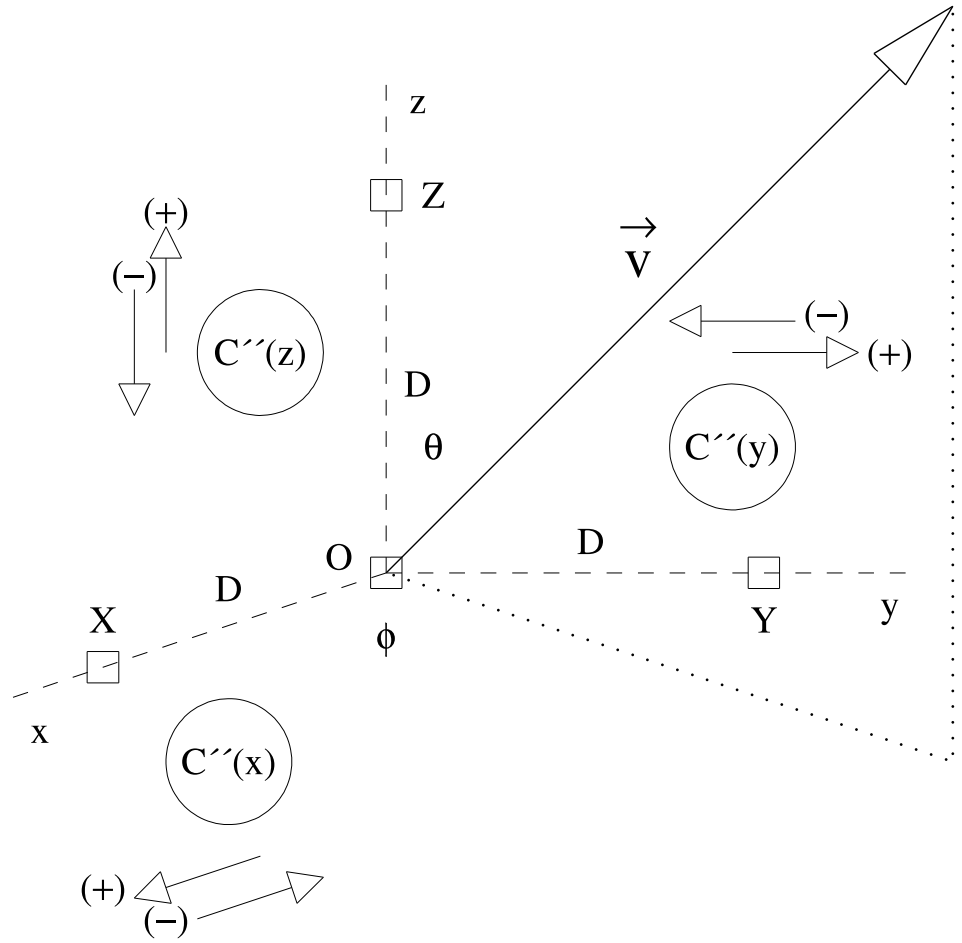


Figure 8: Clock displacements to measure the time intervals  $\Delta T''(\chi, +-)$  ( $\chi = x, y, z$ ) from which the velocity of the travelling inertial frame  $S'$  relative to inertial base frame  $S$ ,  $\vec{v}$ , may be determined. Synchronised clocks (not shown) are located at the marker positions:  $O$ ,  $X$ ,  $Y$  and  $Z$ . See text for discussion.

$$\Delta T''(y, +- ) = -\frac{2Dv}{c^2} \sin \theta \sin \phi. \quad (4.2)$$

$$\Delta T''(x, +- ) = -\frac{2Dv}{c^2} \sin \theta \cos \phi. \quad (4.3)$$

so that (3.7)-(3.9) generalise to

$$v = \frac{c^2}{2D} [\Delta T''(x, +- )^2 + \Delta T''(y, +- )^2 + \Delta T''(z, +- )^2]^{\frac{1}{2}}, \quad (4.4)$$

$$\cos \theta = -\frac{\Delta T''(z, +- )}{[\Delta T''(x, +- )^2 + \Delta T''(y, +- )^2 + \Delta T''(z, +- )^2]^{\frac{1}{2}}}, \quad (4.5)$$

$$\sin \phi = -\frac{\Delta T''(y, +- )}{[\Delta T''(x, +- )^2 + \Delta T''(y, +- )^2]^{\frac{1}{2}}}, \quad (4.6)$$

$$\cos \phi = -\frac{\Delta T''(x, +- )}{[\Delta T''(x, +- )^2 + \Delta T''(y, +- )^2]^{\frac{1}{2}}}. \quad (4.7)$$

For velocities  $v \simeq v_E = 470\text{m/s}$  and distances  $D$  of the order of the circumference of the Earth  $\simeq 4 \times 10^4\text{km}$ , the values of  $\Delta T''$  will be similar than those shown in Fig. 5 —up to several hundred nanoseconds— and so easily measurable by currently available atomic clocks.

## 5 Internal measurement using a moveable clock of the speed of Galileo's ship

A common way of stating the Special Relativity Principle (SRP) is to assert the impossibility, by any physical means whatever, to detect the existence of uniform translational motion by purely internal measurements, This was done in a particularly graphic way by Galileo in his book 'Dialogue Concerning the Two Chief World Systems —Ptolemaic and Copernican' [23] in the following (abridged) passage:

'Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speeds to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; . . . When you have observed all these things carefully . . . have the ship proceed with any speed you like, so long as the speed is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. . . . The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in the water will swim toward the front of their bowl with no more effort than toward the back and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally butterflies and flies will continue their flights indifferently

toward every side, nor will it happen that they are concentrated toward the stern, as if tired from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air'.

The assertion of this passage that the laws of physics are the same in different inertial frames is a valid one for mechanical processes —indeed it follows as a necessary consequence of the invariance of Newton's Second Law of Motion under the Galilean transformations:  $x' = x - vt$ ,  $t' = t$ . However, as will now be demonstrated, it is sufficient to have synchronised precision clocks at different positions in the ship, together with another one that can be moved about the ship at constant speed, to measure, internally, the speed of the ship relative to the surface of the Earth, as well as the local speed of motion of the surface of the Earth in the inertial frame S. As in the previous examples, if time intervals registered by the moving clock during its displacements were directly observable, this clock alone would suffice to perform the experiment.

A precision clock,  $C'''$ , with proper frame  $S'''$ , recording time  $t'''$ , moveable at constant speed  $v''$  in the proper frame  $S''$  of the ship, that may be compared with synchronised clocks  $C''(i)$ , recording time  $t''$ , at different positions,  $i$ , in the ship, is introduced. It is assumed that the ship always moves parallel to a line joining the stern of the ship to its bow, and that  $C'''$  is displaced along this same line, where it is compared with clocks  $C''(1)$  and  $C''(2)$  which are a distance  $D$  apart, say at the stern and bow, respectively, of the ship. The calculation of the speed  $v'$  of the ship (which is assumed to move at an angle  $\theta$  relative to the local line of fixed latitude) in the local comoving rest frame  $S'$  of the Earth's surface, is a straightforward generalisation of the one presented in Section 3 above. The TD effect for the clocks  $C''$  and  $C'''$  is given by the equations:

$$dt = \gamma(C'')dt'' = \gamma(C''')dt''' \quad (5.1)$$

Analogously to Eq. (2.5):

$$\gamma(C''', \pm) = \gamma(C')[\gamma'(C''', \pm) + \beta(C')\beta'(C''', \pm)\gamma'(C''', \pm) \cos \theta] \quad (5.2)$$

where

$$\gamma'(C''', \pm) = \gamma'(C'')[\gamma''(C''') \pm \beta'(C'')\beta''(C''')\gamma''(C''')] \quad (5.3)$$

and

$$\beta'(C''', \pm)\gamma'(C''', \pm) = \gamma'(C'')[\pm\beta''(C''')\gamma''(C''') + \beta'(C'')\gamma''(C''')] \quad (5.4)$$

with the + (−) signs corresponding to motion of  $C'''$  from stern to bow (bow to stern). Using (5.3) and (5.4) to eliminate  $\gamma'(C''', \pm)$  and  $\beta'(C''', \pm)\gamma'(C''', \pm)$  from (5.2) gives:

$$\gamma(C''', \pm) = \gamma(C')\gamma'(C'')\gamma''(C''')\{1 \pm \beta'(C'')\beta''(C''') + \beta(C')[\beta'(C'') \pm \beta''(C''')]\cos \theta\} \quad (5.5)$$

Combining this equation with (2.5) and (5.1):

$$dt'''(\pm) = \frac{\gamma(C'')dt''}{\gamma(C''', \pm)} = \frac{(1 + \beta(C')\beta'(C'') \cos \theta)dt''}{\gamma''(C''')\{1 \pm \beta'(C'')\beta''(C''') + \beta(C')[\beta'(C'') \pm \beta''(C''')]\cos \theta\}} \quad (5.6)$$

If  $T'''(\theta, +)$  and  $T'''(\theta, -)$  are the time intervals recorded by  $C'''$  for passages from stern to bow and from bow to stern respectively, then since the corresponding time interval

$T''$  recorded by  $C'''(1)$  or  $C'''(2)$  is  $D/v''$ , (5.6) gives, on retaining only  $O(\beta^2)$  terms, the equations analogous to (3.1)–(3.4):

$$\Delta T''(\theta, +) \equiv T'''(\theta, +) - T'' = \frac{D}{v''} \left[ -\frac{1}{2} \left( \frac{v''}{c} \right)^2 - \frac{v''(v' + v \cos \theta)}{c^2} \right], \quad (5.7)$$

$$\Delta T''(\theta, -) \equiv T'''(\theta, -) - T'' = \frac{D}{v''} \left[ -\frac{1}{2} \left( \frac{v''}{c} \right)^2 + \frac{v''(v' + v \cos \theta)}{c^2} \right] \quad (5.8)$$

from which follows, analogously to (3.5) or (3.6):

$$\Delta T'''(\theta, +- ) \equiv \Delta T''(\theta, +) - \Delta T''(\theta, -) = -\frac{2D}{c^2}(v' + v \cos \theta) \quad (5.9)$$

If  $v' = 0$  then (3.5) is recovered from (5.9) and  $\Delta T'''$  measures  $v \cos \theta$  so that if  $\theta$  is known  $v$  may be determined. Note that  $\Delta T'''$  is independent of  $v''$ , the speed of the moving clock  $C'''$ . In order to measure  $v'$  directly from (5.9) it is necessary that  $\theta = 90^\circ$  so that it is known that the ship moves directly N–S, or S–N, i.e. along a line of fixed longitude.

A procedure to measure all three unknown quantities  $v'$ ,  $v$  and  $\theta$  in (5.9) is to perform three separate measurements of  $\Delta T'''$  first according to (5.9) and then on changing the direction of motion of the ship by two successive anti-clockwise rotations of  $120^\circ$  each. This gives the following three relations:

$$\Delta T'''(1) \equiv \Delta T'''(\theta, +- ) = -\frac{2D}{c^2}(v' + v \cos \theta), \quad (5.10)$$

$$\Delta T'''(2) \equiv \Delta T'''(\theta + 120^\circ, +- ) = -\frac{2D}{c^2}(v' - vS \cos \theta - vC \sin \theta), \quad (5.11)$$

$$\Delta T'''(3) \equiv \Delta T'''(\theta + 240^\circ, +- ) = -\frac{2D}{c^2}(v' - vS \cos \theta + vC \sin \theta) \quad (5.12)$$

where

$$S \equiv \sin 30^\circ = 1/2, \quad C \equiv \cos 30^\circ = \sqrt{3}/2.$$

These simultaneous equations may be solved to give  $v'$ ,  $v$  and  $\theta$ . It is found that:

$$v' = -\frac{c^2[2S\Delta T'''(1) + \Delta T'''(2) + \Delta T'''(3)]}{4D(1 + S)}, \quad (5.13)$$

$$v = \sqrt{v_S^2 + v_C^2}, \quad (5.14)$$

$$v_S \equiv v \sin \theta = -\frac{c^2[\Delta T'''(3) - \Delta T'''(2)]}{4DC}, \quad (5.15)$$

$$v_C \equiv v \cos \theta = -\frac{c^2[2\Delta T'''(1) - \Delta T'''(2) - \Delta T'''(3)]}{4D(1 + S)}. \quad (5.16)$$

If the direction of motion of the ship relative to the local line of fixed latitude is known, two measurements of  $\Delta T'''$  suffice to determine both  $v'$  and  $v$ , provided the ship does not move along a line of fixed longitude. For example, for West to East and East to West motion, corresponding to a change of  $180^\circ$  in the direction of motion of the ship, (5.9) gives:

$$\Delta T'''(\text{W} - \text{E}, +- ) = -\frac{2D}{c^2}(v' + v), \quad (5.17)$$

$$\Delta T'''(\text{E} - \text{W}, +- ) = -\frac{2D}{c^2}(v' - v). \quad (5.18)$$

So that

$$v' = -\frac{c^2[\Delta T'''(W - E, +-)+\Delta T'''(E - W, +-)]}{4D}, \quad (5.19)$$

$$v = -\frac{c^2[\Delta T'''(W - E, +-)-\Delta T'''(E - W, +-)]}{4D}. \quad (5.20)$$

Assuming  $D = 200\text{m}$  and equatorial motion so that  $v = 470\text{m/s}$ , it is found that, since  $v' \ll v$ , that

$$-\Delta T'''(W - E, +-)\simeq \Delta T'''(E - W, +-)\simeq 2ps$$

If the clock  $C'''$  is moved at walking pace (5 km/h) from the stern of the ship to the bow and *vice versa* the time recorded by  $C'''(1)$  and  $C'''(2)$  during a single clock transport is 144s. The corresponding time uncertainty for time measurement by a clock with the current stability of 100ps/day is 0.17ps, which gives an 8.5% fractional error in the measurement of  $\Delta T'''$  or of  $v$ . If  $v' = 60\text{km/h}$  (16.7m/s) then  $v'/v = 0.035$ . Therefore a one order of magnitude improvement in clock stability would be required to detect  $v'$  at the four standard deviation level, and two orders of magnitude improvement to measure  $v'$  with a relative precision of 0.2%. Such an experiment, which seems within the bounds of experimental possibility, would then falsify, in a convincing manner, the application in the real world of the SRP as a statement of the impossibility, by any physical means whatever, to detect by purely internal measurements, the existence of uniform translational motion<sup>6</sup>. Introducing the name ‘IDUTM’ for this expression of the SRP, where the letters of the acronym stand for ‘(I)mpossibility to (D)etect (U)niform (T)ranslational (M)otion’, then the correct calculation of time dilation, already verified by the HKE, itself predicts, as shown above, violation of IDUTM. Thus special relativity itself predicts violation of this expression of the SRP as applied to *all* (as opposed to purely mechanical) physical measurements.

## 6 Summary and Conclusions

The good agreement obtained, in the HKE, between the theoretical predictions and experimental results for SR (TD) effects demonstrates the importance of correct definitions of initial conditions in an experiment where TD is observed. The crucial concepts are those

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<sup>6</sup>Because, however, Newton’s Second Law of motion, in the form  $d\vec{p}/dt = \vec{F}$  with suitably defined relativistic definitions of momentum,  $\vec{p}$ , and force,  $\vec{F}$ , holds in all inertial frames, the SRP retains its validity in special relativity for all *mechanical* experiments, and it remains true that the results of such experiments are unable to detect, internally, the presence of uniform translational motion. Galileo’s assertion concerning mechanical experiments therefore remains valid in special relativity. A ‘mechanical experiment’ is defined in the present paper as one in which Newton’s Second Law, or some theoretically equivalent concept, such as Hamilton’s Principle, is essential for its analysis.

of ‘coordinate time’ [3, 4] or the equivalent ones of ‘base’ and ‘travelling’ frames [5, 6]. The ‘base frame’, in which ‘coordinate time’ is recorded by a clock at rest, is an inertial frame in which the velocities of one or more ‘travelling’ frames participating in a TD experiment are specified. The TD effect is asymmetric between base and travelling frames [5]. Clocks in a travelling frame appear to run slow when viewed from their base frame, whereas clocks in the base frame appear to run fast when viewed from an associated travelling frame. An immediate corollary of this, as demonstrated by the SR effects observed in the HKE, is that *The TD effect observed between clocks in two travelling frames does not depend only on the relative velocity of the two frames.* An example of this is shown in Fig. 6, where different moving clocks, with the same absolute relative velocity, may be observed to run either slower or faster than one at rest in a particular travelling frame.

It was incorrectly stated in [3] that this breakdown of the naive dependence of TD on only the relative velocity of the two clocks concerned, in the HKE, was a consequence of the fact that only the hypothetical clock recording coordinate time is in an inertial frame, whereas the airborne and Earth-bound clocks are situated in frames subjected to acceleration. However, it was clearly demonstrated in an experiment performed at CERN shortly after the HKE [13] that the TD effect in muonic clocks in uniform circular motion, subjected to very large transverse accelerations, depends only on the magnitude of the velocity of the clocks, and so is the same as in a travelling inertial frame with the same velocity. This opens the possibility, as described in the present paper, to detect the motion of travelling inertial frames by purely internal observations of time differences between clocks at rest, or in uniform motion, in such frames.

How the angular velocity,  $\vec{\Omega}$ , of the Earth may be determined, in this way, by comparing time intervals recorded by an aircraft, following Great Circle routes at constant speed, with those recorded by a clock at rest on the surface of the Earth, is described in Section 2. Interestingly enough, the speed of the aircraft is required only to measure the Earth’s radius, from the circumnavigation period. The time-difference formula (2.14) used to determine the speed of an equatorial point on the Earth’s surface is independent of the speed of the aircraft, provided that the latter is much smaller than  $c$ . In Section 3 it is shown how the local velocity of a point on the Earth’s surface may be determined from the differences of time intervals recorded by a movable clock and an array of synchronised clocks, at known fixed locations. The synchronised clocks are introduced only for experimental convenience. Since the time interval  $T'$  recorded by a clock at rest in the inertial frame, the velocity of which is being measured, cancels in the double time difference  $\Delta T''(+ -)$  a single unsynchronised moveable clock is sufficient, in the ideal case, to perform all the experiments proposed. Section 4 generalises this method to determine the frame velocity vector of an arbitrary travelling frame in free space. In the case that the magnitude of the velocity vanishes the inertial frame is a base frame [5, 6].

Finally, in an evident generalisation of the methods used in the previous sections, it is shown, in Section 5, how both the velocity of a ship in uniform motion and the local velocity of adjacent points on the Earth may be derived from the differences between time intervals recorded by synchronised clocks at fixed positions on the ship and by a uniformly moving clock, or, in an ideal experiment, between time intervals recorded only by the moving clock. The possibility to determine, in this way, from purely internal measurements, the speed of the ship evidently falsifies the applicability to this problem of the

Special Relativity Principle, as the statement that such a measurement is impossible, by any internal physical measurements whatever. Although such a statement (IDUTM) does not hold when observations of the TD effect of special relativity are invoked, it remains universally valid for measurements of mechanical processes, since Newton's Second Law is covariant under both Galilean and Lorentz transformations. Galileo's assertion of the impossibility, by any mechanical means, to detect uniform translational motion, remains then valid in special relativity.

Since TD is an  $O(\beta^2)$  effect that vanishes in the limit  $c \rightarrow \infty$ , IDUTM does hold universally (i.e. for both mechanical and non-mechanical experiments) in Galilean relativity. This is because all the double time differences, on which the analyses presented in the present paper depend, vanish in the Galilean limit where  $c \rightarrow \infty$ . Thus, IDUTM is valid as applied to mechanical experiments in both Galilean and special relativity, and also to non-mechanical experiments in Galilean relativity, but fails, as discussed in the present paper, for non-mechanical experiments in special relativity.

Since writing the original version of this paper it has come to my attention that internal measurement of uniform translational motion is also possible by exploiting the classical Sagnac effect [18, 19].

### **Acknowledgments**

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## Appendix A

The unit vector  $\hat{t}$ , tangent to the circle of fixed latitude, in the W–E direction, at any point on the Great Circle followed by the aircraft (see Figs. 1 and 2), can be written in primed coordinates as:

$$\hat{t}_{x'} = -\sin \phi', \quad (\text{A.1})$$

$$\hat{t}_{y'} = \cos \phi', \quad (\text{A.2})$$

$$\hat{t}_{z'} = 0 \quad (\text{A.3})$$

where  $\phi'$  is the azimuthal angle measured from the line segment QN, (see Fig. 1) in the plane of a circle of fixed latitude  $\lambda$ .

Transforming  $\hat{t}$  into the unprimed system according to:

$$x = x', \quad (\text{A.4})$$

$$y = y' \cos \psi + z' \sin \psi, \quad (\text{A.5})$$

$$z = z' \cos \psi - y' \sin \psi \quad (\text{A.6})$$

gives

$$\hat{t}_x = -\sin \phi', \quad (\text{A.7})$$

$$\hat{t}_y = \cos \phi' \cos \psi, \quad (\text{A.8})$$

$$\hat{t}_z = -\cos \phi' \sin \psi. \quad (\text{A.9})$$

Since the projection of  $\hat{t}$  into the  $xy$  plane (the plane of the Great Circle around which the aircraft moves) is tangent to the Great Circle at P (see Fig. 2a):

$$\frac{\hat{t}_x}{\hat{t}_y} = -\tan \phi = -\frac{\tan \phi'}{\cos \psi} \quad (\text{A.10})$$

so that

$$\tan \phi' = \cos \psi \tan \phi. \quad (\text{A.11})$$

Using (A.11) to eliminate  $\phi'$  from (A.7)-(A.9) gives:

$$\hat{t}_x = -\frac{\cos \psi \tan \phi}{(1 + \cos^2 \psi \tan^2 \phi)^{\frac{1}{2}}}, \quad (\text{A.12})$$

$$\hat{t}_y = \frac{\cos \psi}{(1 + \cos^2 \psi \tan^2 \phi)^{\frac{1}{2}}}, \quad (\text{A.13})$$

$$\hat{t}_z = -\frac{\sin \psi}{(1 + \cos^2 \psi \tan^2 \phi)^{\frac{1}{2}}}. \quad (\text{A.14})$$

Since

$$(\vec{v}'_A)_x = -v'_A \sin \phi, \quad (\text{A.15})$$

$$(\vec{v}'_A)_y = v'_A \cos \phi, \quad (\text{A.16})$$

$$(\vec{v}'_A)_z = 0. \quad (\text{A.17})$$

it follows that

$$\begin{aligned}\cos \theta &= \frac{\hat{t} \cdot \vec{v}'_A}{|\vec{v}'_A|} = \frac{\cos \psi \tan \phi \sin \phi + \cos \psi \cos \phi}{(1 + \cos^2 \psi \tan^2 \phi)^{\frac{1}{2}}} \\ &= \frac{\cos \psi}{(1 - \sin^2 \psi \sin^2 \phi)^{\frac{1}{2}}}\end{aligned}\quad (\text{A.18})$$

which is Eq. (2.1) of the main text. Eq. (2.2) generalises, for an arbitrary point on the Great Circle, to:

$$\sin \lambda = \sin \psi \sin \phi. \quad (\text{A.19})$$

Eliminating  $\phi$  between (A.18) and (A.19) gives:

$$\cos \psi = \cos \theta \cos \lambda \quad (\text{A.20})$$

which is the generalisation of Eq. (2.9) of the text for an arbitrary point on the Great Circle.

## Appendix B

Taking into account both SR and GR effects, for a clock in motion on the surface of the Earth, assumed to be spherical, of radius  $R$ , and of mass,  $M_E$ , an increment,  $d\tau$ , of the proper time of the clock is related to the corresponding increment,  $dt$  of coordinate time, as discussed in Section 2, by the Schwarzschild metric equation [24, 25]

$$d\tau = \left[ \left( 1 + \frac{2\phi_E}{c^2} \right) - \frac{1}{c^2} \left( \frac{v_r^2}{1 + \frac{2\phi_E}{c^2}} + v_\theta^2 + v_\phi^2 \right) \right]^{\frac{1}{2}} dt \quad (\text{B.1})$$

where  $v_r$ ,  $v_\theta$  and  $v_\phi$  are components of the velocity of the clock in the ECI frame in a polar coordinate system with origin at the center of the Earth, polar axis oriented in the S–N direction and  $\phi_E = -GM_E/R$  is the gravitational potential at the surface of the Earth. For a clock undergoing equatorial circumnavigation,  $v_r = v_\theta = 0$  and  $v_\phi = v_E = c\beta_E$ . Denoting, as in the text, the proper time of a clock by  $t'$  or  $t''$ , Eq. (B.1) then simplifies, for clocks at rest on the surface of the Earth, undergoing W–E (+) or E–W (–) equatorial round trips at speed  $v'_A = c\beta'_A$  relative to the surface of the Earth, to the respective equations:

$$dt' = \left[ \left( 1 + \frac{2\phi_E}{c^2} \right) - \beta_E^2 \right]^{\frac{1}{2}} dt, \quad (\text{B.2})$$

$$dt''_{\pm} = \left[ \left( 1 + \frac{2\phi_E}{c^2} \right) - \beta_A(\pm)^2 \right]^{\frac{1}{2}} dt \quad (\text{B.3})$$

where

$$\beta_A(\pm) \equiv \frac{\beta_E \pm \beta'_A}{1 \pm \beta_E \beta'_A} \quad (\text{B.4})$$

Taking the quotient of (B.3) and (B.2) gives

$$dt''_{\pm} = \left[ \frac{\left( 1 + \frac{2\phi_E}{c^2} \right) - \beta_A(\pm)^2}{\left( 1 + \frac{2\phi_E}{c^2} \right) - \beta_E^2} \right]^{\frac{1}{2}} dt'. \quad (\text{B.5})$$

Substituting for  $\beta_A(\pm)$  in (B.5) from (B.4) gives, after some algebraic manipulation,

$$dt''_{\pm} = \frac{\sqrt{1 - (\beta'_A)^2}}{1 \pm \beta_E \beta'_A} \left[ \frac{1 - \beta_E^2 + \frac{(1 \pm \beta_E \beta'_A)^2 2\phi_E}{1 - (\beta'_A)^2 c^2}}{1 - \beta_E^2 + \frac{2\phi_E}{c^2}} \right]^{\frac{1}{2}} dt'. \quad (\text{B.6})$$

Setting  $\phi_E = 0$ , Eq. (2.7) for the case  $\lambda = 0$ ,  $\theta = 0$  or  $\pi$  is recovered. Retaining only the first order term in  $\phi_E$  in the power expansion of the terms in the large square bracket of (B.6) gives:

$$dt''_{\pm} = \left\{ \frac{\sqrt{1 - (\beta'_A)^2}}{1 \pm \beta_E \beta'_A} + \frac{\phi_E}{(1 - \beta_E^2)c^2} \left[ \frac{1 \pm \beta_E \beta'_A}{\sqrt{1 - (\beta'_A)^2}} - \frac{\sqrt{1 - (\beta'_A)^2}}{1 \pm \beta_E \beta'_A} \right] \right\} dt'. \quad (\text{B.7})$$

while retaining only  $O(\beta^2)$  terms on the right side of (B.7) gives:

$$dt''_{\pm} = \left\{ 1 - \frac{(\beta'_A)^2}{2} \mp \beta_E \beta'_A + \frac{\phi_E}{c^2} [(1 + \beta_E^2 + \beta'_A)^2 \pm 2\beta_E \beta'_A] \right\} dt'. \quad (\text{B.8})$$

Integrating over the W–E or E–W round trips to give:  $T''(+)$   $\equiv \int dt''_+$ ,  $T''(-)$   $\equiv \int dt''_-$  and  $T' \equiv \int dt'$  it is found that

$$\Delta T''(\pm) = T''(+)$$
  $- T''(-) = -2T' \beta_E \beta'_A (1 - \frac{2\phi_E}{c^2}) + O(\phi_E^2)$   $\quad (\text{B.9})$

or, using (2.12)

$$\Delta T''(\pm) = -\frac{4\pi v_E R}{c^2} (1 - \frac{2\phi_E}{c^2}) + O(\phi_E^2) \quad (\text{B.10})$$

So, on including the  $O(\phi_E)$  GR correction, Eq. (2.15) is modified to:

$$|\vec{\Omega}| = \frac{v_E}{R} = c^2 \frac{|\Delta T''(\theta_0 = 0, \lambda_0 = 0, +-)|}{4\pi R^2} (1 + \frac{2\phi_E}{c^2}). \quad (\text{B.11})$$

## Appendix C

Consider a transmitter and receiver of light signals on the Earth's Equator, separated by a distance  $\Delta s \ll R$ . The coordinate time interval,  $\Delta t$ , between emission and reception of light signals, assuming that the receiver is placed to the east of the transmitter, is given by the equation:

$$c\Delta t = \Delta s + v_E \Delta t \quad (\text{C.1})$$

from which it follows that

$$\Delta t(\beta_E) = \frac{\Delta s}{c(1 - \beta_E)} \quad (\text{C.2})$$

where  $v_E$  of the speed of the transmitter or receiver in the ECI frame and  $\beta_E \equiv v_E/c$ . The time dilation equation

$$\Delta t(\beta_E) = \gamma(\beta_E) \Delta \tau(\beta_E) \quad (\text{C.3})$$

gives the corresponding time interval recorded by a clock at rest at the Equator:

$$\Delta \tau(\beta_E) = \frac{\Delta s}{c\gamma(\beta_E)(1 - \beta_E)} = \frac{\Delta s}{c} \sqrt{\frac{1 + \beta_E}{1 - \beta_E}}. \quad (\text{C.4})$$

Denoting, as in the text, the proper time of a clock at rest on the surface of the Earth by  $t'$ , and considering a light signal that performs a W–E equatorial circumnavigation of the Earth, so as to arrive back at the transmitter, integrating (C.4) gives:

$$t'(\beta_E) = \int d\tau(\beta_E) = \frac{2\pi R}{c} \sqrt{\frac{1+\beta_E}{1-\beta_E}}. \quad (\text{C.5})$$

The ‘Sagnac Effect’ [17, 26] in the comoving frame of the transmitter is then defined as the time interval:

$$T'_S(\beta_E) \equiv t'(\beta_E) - t'(\beta_E = 0) = \frac{2\pi R}{c} \left[ \sqrt{\frac{1+\beta_E}{1-\beta_E}} - 1 \right]. \quad (\text{C.6})$$

Retaining terms up to  $O(\beta^3)$  in the power expansion of the square bracket in (C.6) gives

$$T'_S(\beta_E) = \frac{2\pi R\beta_E}{c} \left[ 1 + \frac{\beta_E}{2} + \frac{\beta_E^2}{2} \right] + O(\beta_E^4). \quad (\text{C.7})$$

For E–W circumnavigation the same formula is valid with the replacement  $\beta_E \rightarrow -\beta_E$ . Defining a double time interval difference  $\Delta T'_S(+ -)$ , it is found that

$$\Delta T'_S(+ -) \equiv T'_S(\beta_E) - T'_S(-\beta_E) = \frac{4\pi R\beta_E}{c} \left[ 1 + \frac{\beta_E^2}{2} \right] + O(\beta_E^5). \quad (\text{C.8})$$

This formula may be compared with (2.14) for the case  $\theta_0 = \lambda_0 = 0$ :

$$\Delta T''(\theta_0 = \lambda_0 = 0, + -) = -\frac{4\pi R\beta_E}{c} \left[ 1 - \frac{(\beta'_A)^2}{2} \right] + O(\beta^5) \quad (\text{C.9})$$

where up to  $O(\beta^3)$  terms in the expansion of (2.7) have been retained. Note that, at this order,  $\Delta T''(+ -)$  is no longer independent of  $v'_A$ . It can be seen from (C.8) and (C.9) that, on neglecting  $O(\beta^3)$  and higher order terms in both equations that:

$$\Delta T'_S(+ -) = -\Delta T''(\theta_0 = \lambda_0 = 0, + -) = \frac{4\pi Rv_E}{c^2}. \quad (\text{C.10})$$

This approximate equality has led to the unfortunate circumstance of conflating in the literature [7, 14, 15] the asymmetrical time dilation effect observed in the HKE with the Sagnac Effect, that, at the order of approximation at which Eq. (C.10) is valid, is a purely classical one [27] implicating neither SR nor GR —it follows from the definition of relative velocity of two objects in a single frame of reference, which is the same in Galilean relativity and SR. It is not clear why this misleading lack of distinction between two completely different physical phenomena should have occurred. The experiments in which  $\Delta T'_S(+ -)$  and  $\Delta T''(+ -)$  are defined are not the same. The quantity  $\Delta T'_S(+ -)$  refers to differences between the times of reception of round-trip light signals, in the W–E or E–W directions, on the rotating Earth. The quantity  $\Delta T''(+ -)$  is the difference of the time intervals recorded by a clock moving (with arbitrary uniform velocity  $\ll c$ ) relative to the surface of the Earth, for W–E and E–W round trips. Comparison of (C.8) and (C.9) shows that not only is the sign of the two time interval differences not the same for similar directions of motion of the light signal and the clock, but the  $O(\beta^2)$  corrections are of opposite sign and depend on different quantities:  $\beta_E$  for  $\Delta T'_S(+ -)$  and  $\beta'_A$  for  $\Delta T''(+ -)$ . The Sagnac experiment and the HKE both involve time intervals and the velocity parameter  $v_E$ , but they are physically quite distinct. Indeed some other name should be found for the purely special relativistic, asymmetric, time dilation effect observed for the first time in the HKE and embodied in Eq. (2.15). The Hafele-Keating effect?

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