The Sagnac effect and transformations of relative velocities between inertial frames

J.H.Field
Département de Physique Nucléaire et Corpusculaire, Université de Genève
24, quai Ernest-Ansermet CH-1211Genève 4.
E-mail: john.field@cern.ch

Abstract

The Sagnac effect is analysed in both Galilean and Special relativity within a space-time geometrical model previously developed by Langevin and Post. The effect arises because of the different velocities of different light signals relative to the interferometer. The appropriate relativistic relative velocity transformation formulas obtained differ from the velocity transformation formulas of conventional Special relativity, the latter actually predicting that the Sagnac effect vanishes. The Michelson-Morley experiment is analysed using the same model and a non-vanishing fringe shift, albeit below the sensitivity of all such experiments performed to date, is predicted. The Sagnac effect for neutrinos of the CERN CNGS beam is also discussed. The Sagnac effect indicates that the ECI (Earth Centered Inertial) frame is a preferred one in which light signals have a speed close to c, in the vicinity of the Earth, as predicted by General relativity.

PACS 03.30.+p

Sagnac published the results of his rotating interferometer experiment in 1913 [1]. The principle of the experiment is shown in Fig. 1. Light from a source S is split into two beams by a half-silvered mirror HSM. With the aid of the corner mirrors M_1, M_2, M_3 the light beams return to HSM via clockwise (HSM M_1 M_2 M_3 HSM) or anti-clockwise (HSM M_3 M_2 M_1 HSM) routes where they are combined into a single beam which is observed at D. When the whole apparatus, including the light source and the detector (which in Sagnac's original experiment was a photographic plate) is rotated a fringe shift ΔZ is observed, corresponding, at lowest order in the angular velocity, to a phase difference between the counter-rotating beams of: $\Delta\phi = 2\pi\Delta Z = 8\pi\vec{\Omega}\cdot\vec{A}/(\lambda_0 c)$ where $\vec{\Omega}$ is the angular velocity vector, λ_0 is the vacuum wavelength of the light, $|\vec{A}|$ is the area enclosed by the circulating light beams and $\vec{A}$ is perpendicular to the plane of the interferometer. This phase shift formula, for the case when $\vec{A}$ is parallel to $\vec{\Omega}$ and the axis of rotation passes through the center of a square interferometer, is derived below, from considerations of space-time geometry, in both Galilean and Special relativity.

It is interesting to note that, although Einstein had declared the luminiferous aether to be 'superfluous' in 1905 [2], the title of Sagnac's paper was 'L'éther lumineux démontré par l'effect du vent relatif d'éther dans un interférometer en rotation uniform', or, in English: 'Demonstration of the existence of the luminiferous aether by an aether wind effect in a rotating interferometer'. It was to search for just such an 'aether wind' that the Michelson-Morley experiment [3] and its successors [4, 5, 6, 7, 8, 9] were performed, with negative results in almost all cases. As discussed below, this is not because an 'aether wind' does

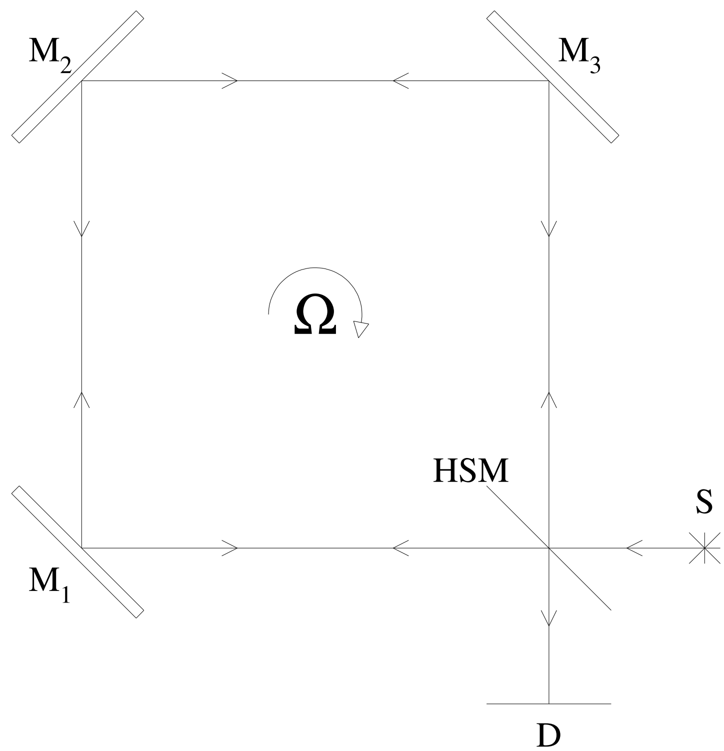

Figure 1: *A Sagnac interferometer. Light signals from a source S are split by the half-silvered-mirror HSM into two beams which follow clockwise (HSM M_1 M_2 M_3 HSM) or anti-clockwise (HSM M_3 M_2 M_1 HSM) paths, of equal length, back to HSM where they are recombined and detected at D. When the interferometer is rotated with angular velocity Ω, a phase shift develops between clockwise- and anti-clockwise-rotating beams due to different times-of-passage of the light signals. The latter result from different velocities of clockwise- and anti-clockwise-rotating light beams relative to the interferometer.*

not exist, but because the corresponding phase shift is of order $(v/c)^2$ to be compared with order v/c for the observed phase shift in Sagnac's interferometer. Sagnac's experiment was repeated, with much improved precisison, by Pogany [10] and especially by Michelson and Gale [11] who used the effect to measure the speed of rotation of the Earth. More recently a related experiment —the ring laser— where counter-rotating laser beams have different characteristic frequencies when the device is rotated [12] was demonstrated and the Sagnac experiment itself was repeated using fibre optic light guides which enabled the development of highly sensitive fibre-optic gyroscopes [13, 14, 15] to detect rotation. Even more recently, it has been shown [16, 17], also by the use of fibre-optic interferometers, that relative translational motion also results in a phase shift due to the same space-time geometrical effect that underlies the original Sagnac experiment. The phase shift for translational motion is [16]: $\Delta\phi = 4\pi L \Delta v/(\lambda_0 c)$ where L is the length of the fibre optic path and Δv the change in the relative velocity of the light signals and the moving interferometer in the laboratory system. The present paper considers only the theory of the original rotating Sagnac experiment, however the essential underlying physics – different velocities of light signals relative to various elements of the interferometer— is the same for all Sagnac-type experiments whether in rotation or uniform translational motion.

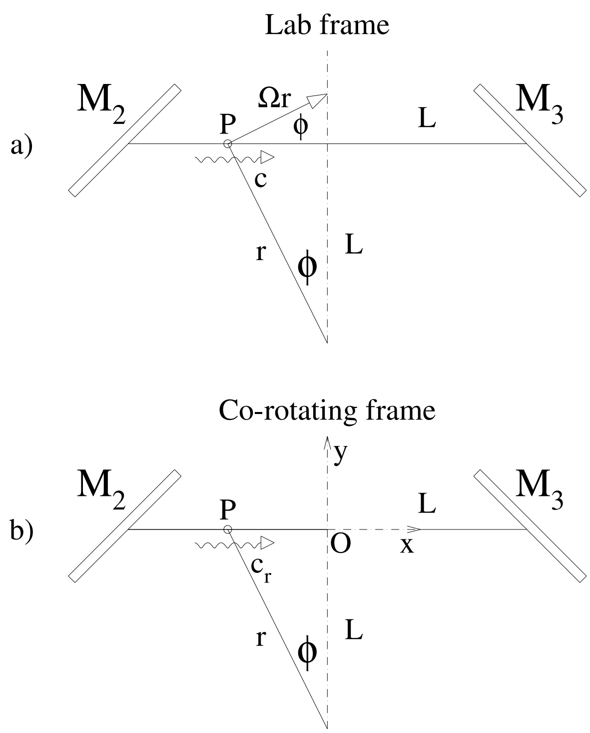

Figure 2: *Space-time geometry in Galilean relativity of the passage of a light signal between end mirrors M_2 and M_3 of the Sagnac interferometer shown in Fig. 1. a) in the laboratory frame; b) in the co-rotating frame of the interferometer. See text for discussion.*

The analysis will be first performed in the context of Galilean relativity, before considering the special relativistic analysis as previously done by Post [18], by suitable modification of a space-time geometrical calculation originally due to Langevin [19].

The fundamental space-time effect underlying the phase shift is a different transit time from beam-splitter to beam-splitter for clockwise- and anticlockwise-rotating beams, when the interferometer is rotating (see Fig. 1). In Fig. 2a a clockwise-moving photon with polar coordinates (r,ϕ) is at position P in the rotating interferometer shown in Fig. 1. The velocity, c_r, of the photon, relative to P, is, from the geometry of Fig. 2a:

$$c_r = c - \Omega r \cos\phi = c - \Omega L. \tag{1}$$

where length of the path of the light signal between mirrors is $2L$. In Galilean relativity this is also the velocity of the photon, relative to P, in the co-rotating frame of the interferometer, where P is at rest, as shown in Fig. 2b. In the co-rotating frame, the photon moves parallel to the x-axis. The time dt_+ to cover an infinitesimal spatial interval dx including P is then:

$$dt_+ = \frac{dx}{c_r} = \frac{r d\phi}{\cos\phi(c-\Omega L)} = \frac{L d\phi}{\cos^2\phi(c-\Omega L)} = \frac{L d(\tan\phi)}{(c-\Omega L)}. \tag{2}$$

Integrating over the range: $-\pi/4 < \phi < \pi/4$ gives

$$t_+ = \frac{L}{(c-\Omega L)} \int_{-1}^{1} d(\tan\phi) = \frac{2L}{(c-\Omega L)}. \tag{3}$$

If T_+ (T_-) is the clockwise (anti-clockwise) flight time of the photon from HSM to HSM, the 4-fold symmetry of the interferometer gives:

$$T_\pm = 4t_\pm = \frac{8L}{(c \mp \Omega L)}. \tag{4}$$

The phase shift due to rotation of the interferometer is then:

$$\begin{aligned} \Delta\phi_{\rm GR} &= 2\pi\nu(T_+ - T_-) = \frac{32\pi\nu\Omega L^2}{c^2(1-\beta(L)^2)} = \frac{8\pi\Omega A \gamma(L)^2}{\lambda_0 c} \\ &= \frac{8\pi\Omega A}{\lambda_0 c}(1+\beta(L)^2) + {\rm O}(\beta(L)^5). \qquad \text{(Galilean relativity)} \end{aligned} \tag{5}$$

where $\beta(L) \equiv \Omega L/c$, $\gamma(L) \equiv 1/\sqrt{1-\beta(L)^2}$ and $A = 4L^2$ is the area enclosed by the circulating light beams. The frequency ν here is that of the source as observed in the co-rotating frame. Since the distances between the source and the various elements of the interferometer are constant there is no classical Doppler effect

In special relativity, time dilation occurs in the comoving inertial frame of the point P of the interferometer, so that, in the co-rotating frame the time interval $dt_\pm$ is replaced by $dt'_\pm = dt_\pm/\gamma(r)$ so that

$$dt'_\pm = \frac{dt_\pm}{\gamma(r)} = \frac{L d\phi}{c\gamma(r)\cos^2\phi(1 \mp \beta(L))}. \tag{6}$$

Then, since $r = L/\cos\phi$,

$$\frac{1}{\gamma(r)} = \frac{\sqrt{1-\alpha^2\tan^2\phi}}{\gamma(L)} \tag{7}$$

where $\alpha \equiv \beta(L)\gamma(L)$. With the aid of the substitution $\alpha \tan\phi = \sin\theta$ Eq. (6) may be integrated to give:

$$T'_{\pm} = 4\int dt'_{\pm} = \frac{4L}{c}\sqrt{\frac{1 \pm \beta(L)}{1 \mp \beta(L)}}\left[\frac{\mathrm{arcsin}\alpha}{\alpha} + \sqrt{1-\alpha^2}\right] \tag{8}$$

and a Sagnac phase shift of:

$$\begin{aligned} \Delta\phi_{\mathrm{SR}} &= 2\pi\nu(T'_{+} - T'_{-}) = \frac{4\pi\Omega A\gamma(L)}{\lambda_0 c}\left[\frac{\mathrm{arcsin}\alpha}{\alpha} + \sqrt{1-\alpha^2}\right] \\ &= \frac{8\pi\Omega A}{\lambda_0 c}\left[1 + \frac{5\beta(L)^2}{3}\right] + \mathrm{O}(\beta(L)^5). \quad \text{(Special relativity)} \end{aligned} \tag{9}$$

The frequency ν here is defined as in Eq. (5) in the case that the source and the HSM are at the same distance from the axis of rotation and so have the same velocity in the laboratory frame. If this is not the case, then the frequency of the light incident on the HSM will be shifted in frequency due to a differential time dilation effect [20, 21].

It is interesting, in view of a comparison with the previously published work of Post [18], to also consider a circular geometry for the interferomenter (see Fig. 8 of Ref [18]) in which the relative velocities of the light signals and the interferometer are given by:

$$c_r^{\pm} = c \mp \Omega R \tag{10}$$

where c_r^{+} (c_r^{-}) are the velocites of clockwise (anticlockwise) rotating light signals, relative to an adjacent point on the interferometer, in the laboratory system, and R the radius of the circular light path. The times-of-passage of the light signals from beam-splitter to beam-splitter in the laboratory system for the counter-rotating signals are:

$$T_{\pm} = \frac{2\pi R}{c_r^{\pm}} = \frac{2\pi R}{c \mp \Omega R}. \tag{11}$$

In Galilean relativity $T_{\pm} = T'_{\pm}$ where $T'_{\pm}$ are the times of passage in the co-rotating frame of the interferometer, so the corresponding Sagnac (S) phase shift is:

$$\begin{aligned} \Delta\phi_{\mathrm{GR}}^{\mathrm{S}} &= 2\pi\nu(T_{+} - T_{-}) = \frac{8\pi^2\nu R\beta(R)}{c(1-\beta(R)^2)} = \frac{8\pi\Omega A\gamma(R)^2}{\lambda_0 c} \\ &= \frac{8\pi\Omega A}{\lambda_0 c}(1 + \beta(R)^2) + \mathrm{O}(\beta(L)^5). \quad \text{(Galilean relativity)} \end{aligned} \tag{12}$$

where $A = \pi R^2$.

In special relativity, due to the time dilation effect, the times-of-passage of light signals are different in the laboratory and co-rotating frames:

$$T'_{\pm} = \frac{T_{\pm}}{\gamma(R)} \tag{13}$$

so that the phase shift becomes:

$$\Delta\phi_{\mathrm{SR}}^{\mathrm{S}} = \frac{8\pi\Omega A\gamma(R)}{\lambda_0 c} = \frac{8\pi\Omega A}{\lambda_0 c}\left[1 + \frac{\beta(R)^2}{2}\right] + \mathrm{O}(\beta(R)^5). \quad \text{(Special relativity)} \tag{14}$$

In view of the time dilation relations (13) the relative velocities of the light signals and the interferometer are not the same in the laboratory and co-rotating systems in the special relativistic case:

$$T'_{\pm} \equiv \frac{2\pi R}{(c_r^{\pm})'} = \frac{T_{\pm}}{\gamma(R)} = \frac{2\pi R}{\gamma(R)[c \mp \Omega R]} = \frac{2\pi R}{\gamma(R) c_r^{\pm}} \qquad (15)$$

so that the relative velocities of the light signals and the interferometer transform between the laboratory and co-rotating frames as

$$(c_r^{\pm})' = \gamma(R) c_r^{\pm} = \gamma(R)[c \mp \Omega R]. \qquad (16)$$

The corresponding formula for angular velocities and clockwise-rotating signals was derived[a] by Post [18]:

$$\omega' = \gamma(R)(\omega - \Omega) \qquad (17)$$

where $\omega' \equiv (c_r^{+})'/R$ and $\omega \equiv c/R$.

The relative velocity transformation formula (16) differs markedly from the relativistic parallel velocity addition relation (RPVAR) [2] which gives:

$$(c^{\pm})' \equiv \frac{c \mp \Omega R}{1 \mp \frac{c(\Omega R)}{c^2}} = c \frac{1 \mp \frac{\Omega R}{c}}{1 \mp \frac{\Omega R}{c}} = c. \qquad (18)$$

The light signals are therefore predicted by the RPVAR to have the same velocity in the co-rotating frame as they do in the laboratory frame. In consequence $T'_{+} = T'_{-} = 2\pi R/c$ so that the Sagnac effect vanishes. For an interferometer with a square configuration as shown in Fig. 1, the velocity vectors of a local point on the interferometer and the light signals are not, in general, parallel and the conventional relativistic velocity addition formula [2] gives:

$$(c^{\pm})' \equiv \frac{c \mp \Omega r \cos\phi}{1 \mp \frac{c(\Omega r \cos\phi)}{c^2}} = c \frac{1 \mp \frac{\Omega r \cos\phi}{c}}{1 \mp \frac{\Omega r \cos\phi}{c}} = c \qquad (19)$$

and again the Sagnac effect vanishes. It is clear from (18) and (19) that the usual relativistic velocity addition formulas are not applicable to the space-time analysis of a Sagnac interferometer.

Writing the RPVAR in terms of scaled velocities $\beta_v \equiv v/c$:

$$\beta_{u'} = \frac{\beta_u - \beta_v}{1 - \beta_u \beta_v} \qquad (20)$$

it is straightforward to show that this equation is mathematically equivalent to [b] either of the formulas:

$$\gamma_{u'} = \gamma_u \gamma_v (1 - \beta_u \beta_v), \qquad (21)$$

$$\gamma_{u'} \beta_{u'} = \gamma_u \gamma_v (\beta_u - \beta_v) \qquad (22)$$

[a] Post actually obtained, by consideration of the geometry of his Fig. 8, and taking into account the time dilation effect in modifying the original Galilean calculation of Langevin [19] the formula $d\phi = d\phi' + \gamma(R)\Omega dt'$. This is Eq. (24) of [18]. From this follows: $d\phi/dt' = d\phi'/dt' + \gamma(R)\Omega$. Time dilation gives $d\phi/dt' = \gamma(R) d\phi/dt$ so that $\omega' = \gamma(R)(\omega - \Omega)$ where $\omega \equiv d\phi/dt$ and $\omega' \equiv d\phi'/dt'$.

[b] That is, by postulating any one of Eqs. (20), (21) and (22) the remaining two may be obtained by purely algebraic manipulation.

where $\gamma_v \equiv 1/\sqrt{1-\beta_v^2}$, which, in turn, are, respectively, equivalent to the transformation relations of relativistic energy: $E = \gamma_v mc^2$, and momentum: $p = \gamma_v mv$. Thus one correct physical interpretation of the RPVAR is to be found in relativistic kinematics rather than in space time geometry. For further discussion of this important point see Refs. [22, 23]. The formula (21) also gives the transformation of the time dilation factor γ between different inertial frames, as exemplified by its application to the Hafele-Keating experiment [24].

The Michelson-Morley (MM) experiment will now be analysed in the same manner as the Sagnac interferometers discussed above, i.e. it will be assumed that the speed of light has the value c in the laboratory frame and the formula (16) will be used to find the relative velocity of the light signals in the rest frame of the interferometer, which is an inertial frame, rather than the uniformly rotating one of a Sagnac interferometer. The appropriate 'laboratory frame' for experiments performed on the surface of the Earth will be discussed below. The analysis of light signals in the transverse arm of a Michelson interferometer is familar from elementary derivations of the time dilation relation [25] — the velocity of the light signals is equal to c, both in the laboratory frame and in the rest frame of the interferometer— so that, if the length of each arm is D, the time-of-passage in the transverse arm in the interferometer rest frame is $T'_{\rm T} = 2D/c$. If v is the velocity of interferometer in the laboratory system, then on making the replacement $\Omega R \to v$ in (16), the time-of-passage in the rest frame of the interferometer of the light signal in the longitudinal arm is:

$$T'_{\rm L} = \frac{D}{c\gamma_v}\left[\frac{1}{1-\beta_v} + \frac{1}{1+\beta_v}\right] = \frac{2D\gamma_v}{c}. \qquad (23)$$

If the interferometer is rotated through 90° around a vertical axis the longitudinal and transverse arms are exchanged resulting in a phase shift proportional to twice the time difference $T'_{\rm L} - T'_{\rm T}$:

$$\Delta\phi_{\rm SR}^{\rm MM} = 2[2\pi\nu(T'_{\rm L} - T'_{\rm T})] = \frac{8\pi D}{\lambda_0}(\gamma_v - 1) = \frac{4\pi D}{\lambda_0}\beta_v^2 + {\rm O}(\beta_v^4). \quad \text{(Special relativity)} \qquad (24)$$

In Galilean relativity the phase shift is, at ${\rm O}(\beta_v^2)$, a factor of two larger. For comparison the phase shift in the Sagnac interferometer of Fig. 1 may be written as:

$$\Delta\phi_{\rm SR}^{\rm S} = \frac{32\pi L}{\lambda_0}\beta(L) + {\rm O}(\beta(L)^3). \qquad (25)$$

The β_v^2 dependence of $\Delta\phi_{\rm SR}^{\rm MM}$ as compared to the $\beta(L)$ dependence of $\Delta\phi_{\rm SR}^{\rm S}$ explains why the Sagnac experiment successfully detected an 'aether wind' on the surface of the Earth while the MM experiment and later improved versions operating on the same principle failed to do so. In the Michelson-Gale Sagnac experiment situated at latitude 41°46′N the value of $\beta(L)$ for the East-West pointing arm of the interferometer due to the rotation of the Earth was $(0.34\ {\rm km/s})/c = 1.1 \times 10^{-6}$. The corresponding Sagnac phase shift was 0.23 of a fringe width. Placing a Michelson interferometer with a similar light source and dimensions $2L \simeq D = 0.5$ km at the same latitude as the Michelson-Gale experiment Eqs. (24) and (25) predict, for the ratio of phase shifts:

$$\frac{\Delta\phi_{\rm SR}^{\rm MM}}{\Delta\phi_{\rm SR}^{\rm S}} = \frac{\beta(L)}{4} = 2.8 \times 10^{-7} \qquad (26)$$

corresponding to a phase shift of 6.3×10^{-8} of a fringe in the Michelson interferometer.

In interpreting the results of the MM experiment and its successors it was usually assumed that the 'aether' was at rest relative to the Solar System which corresponds to a value of β_v in (24) equal the the speed of rotation of the Earth around the Sun of 29.8 km/s so that $\beta_v \simeq 10^{-4}$. This gives a phase shift in a Michelson interferometer 10^4 times larger than a value of β_v corresponding to the rotation of the Earth about its polar axis. The upper limit: $\beta_v \simeq 10^{-5}$ ($v \simeq 10$ km/s) obtained by the Kennedy-Thorndike experiment [7], which has a sensitivity of about 10^{-5} of an interference fringe width, was still some 30 times larger than the velocity of the surface of the Earth in the Michelson-Gale experiment. At least another two orders of magnitude improvement in the sensitivity of a Michelson interferometer would therefore be needed to detect the speed of the 'aether wind' generated by the rotation of the Earth.

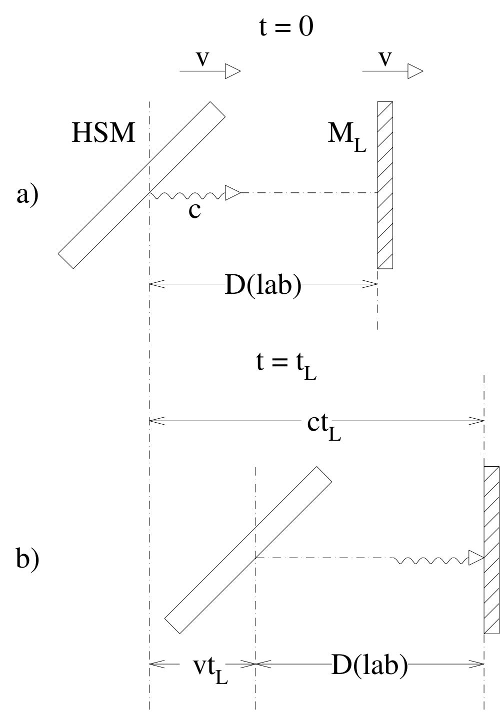

Figure 3: *Space-time geometry of the passage of a light signal in the longitudinal arm of a Michelson interferometer. The latter is at rest on the surface of the Earth with the arm directed in the West-to-East direction. The laboratory frame is the ECI frame and the velocity v of the interferometer is due to the rotation of the Earth. See text for discussion.*

The space-time geometry for the passage of a light signal in the longitudinal arm of a Michelson interferometer between the half-silvered-mirror HSM and the end mirror of the arm, $\mathrm{M_L}$ is shown in Fig. 3. The light signal travels at speed c in the laboratory frame while the velocity v of the interferometer in the West-to-East direction, is due to the rotation of the Earth, i.e. the interferometer is at rest on the surface of the Earth. The light signal leaves HSM at laboratory time $t = 0$ and reaches $\mathrm{M_L}$ when $t = t_{\mathrm{L}}$. The geometry of Fig. 3b gives:

$$ct_{\mathrm{L}} = vt_{\mathrm{L}} + D(\mathrm{lab}) \tag{27}$$

where $D(\mathrm{lab})$ is the separation of HSM and $\mathrm{M_L}$ in the laboratory frame. The speed of the light signal in the rest frame of the interferometer is given by Eq. (16) as:

$$(c_r^+)' = \gamma_v(c - v) \tag{28}$$

so that the time-of-passage of the light signal in the rest frame of the interferometer, t'_{L}, is

$$t'_{\mathrm{L}} = \frac{D}{(c_r^+)'} = \frac{D}{\gamma_v(c-v)} = \frac{D t_{\mathrm{L}}}{\gamma_v D(\mathrm{lab})} \tag{29}$$

where, in the last member, Eq. (27) is used, after transposition, to eliminate $c - v$. Combining the time dilation relation

$$t_{\mathrm{L}} = \gamma_v t'_{\mathrm{L}} \tag{30}$$

with (29) then shows that

$$D(\mathrm{lab}) = D. \tag{31}$$

There is no 'length contraction' effect.

In the conventional interpretation of the MM experiment the failure to observe any phase shift between signals in the longitudinal and transverse arms is assumed to imply that $T'_{\mathrm{L}} = T'_{\mathrm{T}}$. The space-time geometry of the laboratory frame gives a time-of-passage of the light signal from HSM to $\mathrm{M_L}$ and back, in this frame, of:

$$T_{\mathrm{L}} = D(\mathrm{lab})\left[\frac{1}{c-v} + \frac{1}{c+v}\right] = \frac{2D(\mathrm{lab})}{c(1-\beta_v^2)}. \tag{32}$$

Now

$$T'_{\mathrm{T}} = \frac{2D}{c} \tag{33}$$

and time dilation gives:

$$T_{\mathrm{L}} = \gamma_v T'_{\mathrm{L}}. \tag{34}$$

Combining (32), (33) and (34), on the assumption $T'_{\mathrm{L}} = T'_{\mathrm{T}}$ (no phase shift), gives

$$T_{\mathrm{L}} = \gamma_v T'_{\mathrm{L}} = \gamma_v T'_{\mathrm{T}} = \frac{2\gamma_v D}{c} = \frac{2D(\mathrm{lab})}{c(1-\beta_v^2)} \tag{35}$$

from which follows:

$$D(\mathrm{lab}) = \frac{D}{\gamma_v}. \quad (T'_{\mathrm{L}} = T'_{\mathrm{T}}) \tag{36}$$

This is the 'length contraction' effect [26] which explains a null result for the MM experiment in conventional Special relativity. As explained above, for consistency with the observed Sagnac effect, a non-vanishing phase shift must exist in a MM-type experiment, but, to date, no such experiment has had sufficient sensitivity to observe the phase shift. For further discussion of the spurious nature of 'length contraction' and 'relativity of simultaneity' see [22, 23] and references therein.

As described in Refs. [27, 28] corrections for the Sagnac effect are routine in the operation of the Global Positioning System (GPS). The velocity of GPS microwave signals in the rest frame of a GPS receiver are calculated according to the Galilean formula (1) above. Similar corrections are applied in tests, using the GPS, of the isotropy of the speed of light [29]. In this case, as also in the Michelson-Gale experiment, the 'laboratory frame', in which the speed of light is assumed to be c, is the Earth-Centered-Inertial (ECI) frame which is the co-moving inertial frame of the centroid of the Earth with axes pointing to fixed directions on the celestial sphere. It is in this frame that the Earth's gravitational field is given by the Schwartschild metric [30, 31] and which effectively contains the 'aether', relative to which, 'winds' were observed by Sagnac, and Michelson and Gale. It is indeed a prediction of General relativity that, in just this frame, the speed of light is (very nearly) equal to c. 'Very nearly' because of the Shapiro delay [32] of light signals crossing the Earth's gravitational field. For signals from the GPS satellites such delays are less than 200ps [27] and so give no preceptible effect in GPS operation.

The Sagnac effect for neutrinos of the CERN CNGS beam [33] as detected in OPERA [34] in the Gran Sasso Laboratory has recently been considered [35]. Neutrinos, with energies around 17 GeV, from decays of charged pions or kaons are directed in a roughly South-Easterly direction through the crust of the Earth and are detected after a flight distance of about 730 km in the underground detector OPERA. As for photons in the Sagnac and Michelson-Gale experiments, the neutrinos are expected to have speed c in the ECI frame. During the 2.4 ms time-of-flight of the neutrinos the OPERA detector moves a distance 0.835 m [35] in an Easterly direction due to the rotation of the Earth. This increases the time-of-flight of the neutrinos by 2.2 ns [35]. This implies, in turn, that that the neutrinos have an average speed, in the co-moving inertial frame of OPERA (in which the CERN-OPERA separation is constant), that is 9.05×10^{-2} % less than c. Notice that if the CERN neutrino beam were instead directed in a South-Westerly direction the measured speed of the neutrinos would be, by a similar fraction, *greater than* c, so that when the Sagnac efect is taken into account speeds of particles relative to detectors are not limited to be less than or equal to c. Making use of detailed survey information on the positions of the neutrino source and the OPERA detector [36] the angle, α, between the neutrino beam direction and the direction of motion of OPERA in the ECI frame is found to be $\alpha = 37.8°$.

According to the conventional velocity transformation formulas of special relativity [2] the velocity components of the neutrinos, parallel to ($v_{\parallel}$), and perpendicular to ($v_{\perp}$), the direction of motion of OPERA, in the co-moving inertial frame of the latter, when they are assumed to have speed c in ECI frame, are:

$$v_{\parallel} = c\left[\frac{\cos\alpha - \beta_{\rm O}}{1-\beta_{\rm O}\cos\alpha}\right] \tag{37}$$

$$v_{\perp} = c\left[\frac{\sin\alpha}{\gamma_{\rm O}(1-\beta_{\rm O}\cos\alpha)}\right] \tag{38}$$

where $\beta_{\rm O} \equiv v_{\rm O}/c$, $\gamma_{\rm O} \equiv 1/\sqrt{1-\beta_{\rm O}^2}$ and $v_{\rm O} = 323$ m/s is the speed of OPERA in the ECI frame. The speed, v, of the neutrinos in the OPERA frame is then given by:

$$\begin{aligned} v^2 = v_{\parallel}^2 + v_{\perp}^2 &= c^2\left[\frac{(\gamma_{\rm O}^2-1)\cos^2\alpha - 2\beta_{\rm O}\gamma_{\rm O}^2\cos\alpha + \gamma_{\rm O}^2\beta_{\rm O}^2 + 1}{\gamma_{\rm O}^2(1-\beta_{\rm O}\cos\alpha)^2}\right] \\ &= c^2\left[\frac{\gamma_{\rm O}^2\beta_{\rm O}^2\cos^2\alpha - 2\beta_{\rm O}\gamma_{\rm O}^2\cos\alpha + \gamma_{\rm O}^2}{\gamma_{\rm O}^2(1-\beta_{\rm O}\cos\alpha)^2}\right] \\ &= c^2 \end{aligned} \tag{39}$$

(where the identity $\gamma_{\rm O}^2 \equiv \gamma_{\rm O}^2\beta_{\rm O}^2 + 1$ has been used) so that the Sagnac effect vanishes.

The current OPERA measurement [37] of the neutrino time-of-flight gives a value of v significantly greater than c:

$$(v - c)/c = (2.48 \pm 0.28(\mathrm{stat}) \pm 0.3(\mathrm{syst})) \times 10^{-5}$$

This 6.0 σ effect increases to 6.2 σ whan the Sagnac effect (not considered in [37]) is taken into account. In order for the velocity measurement to be sensitive to the Sagnac effect, at least an order of magnitude reduction in the uncertainty of the time-of-flight measurement (currently $\simeq 10ns$) is required. In contrast, the uncertainty of 20 cm in the flight distance given in [36] is about a factor of four less than the displacement due to the Sagnac effect.

The final conclusions are that the ECI frame constitutes a physically-preferred reference system for light signals or neutrinos in the vicinity of the Earth and that the Sagnac effect is not correctly described by the velocity transformation formulas of conventional special relativity. There is clearly an important mismatch between what is known and applied by the engineers of the GPS system, and the content of the scientific literature and text books on Special relativity theory, that needs to be rectified.

References

[1] G. Sagnac, Compt. Rend. **157** 708-710, 1410-1413 (1913).

[2] A. Einstein, Ann. Physik **17**, 891 (1905).
English translation by W. Perrett and G.B. Jeffery in 'The Principle of Relativity' (Dover, New York, 1952) , p.37, or in 'Einstein's Miraculous Year' (Princeton University Press, Princeton, New Jersey, 1998) p.123.

[3] A.A. Michelson and E.W. Morley, Phil. Mag. **24**, 449 (1887).

[4] R.J. Kennedy, Proc. Nat. Acad. Sci. **12** 621 (1926).

[5] K.K. Illingworth, Phys. Rev. **30** 692 (1927).

[6] G. Joos, Ann. Physik **7**, 385 (1930).

[7] R.J. Kennnedy and E.M. Thorndike, Phys. Rev. **42** 400 (1932).

[8] D.C. Miller, Rev. Mod. Phys. **5** 203 (1933).

[9] R.S. Shankland *et al*, Rev. Mod. Phys. **27** 16 (1955).

[10] B. Pogany, Ann. Physik **80**, 217 (1926), **85**, 224 (1928).

[11] A.A. Michelson and H.G. Gale, Astrophys. J. **61**, 137, 140 (1925).

[12] W.M. Macek and D.T.M. Davis, Appl. Phys. Lett. **2**, 67 (1963).

[13] R.A. Bergh, H.C. Lefevre and H.J. Shaw, Optics Lett. **6**, 502 (1981).

[14] J.L. Davis and S. Ezekiel, Optics Lett. **6**, 505 (1981).

[15] ' Sensitive fiber-optic gyroscopes', Physics Today, Oct 1981 p.20.

[16] R. Wang *et al*, Physics Letters **A 312**, 7-10 (2003).

[17] R. Wang, Y. Zheng and A. Yao, Phys. Rev. Lett. **93** 143901 (2004).

[18] E.J. Post, Rev. Mod. Phys. **39** No 2 475-493 (1967).

[19] P. Langevin, Compt. Rendu. **173** 831 (1921), **205** 51 (1937).

[20] L.B. Okun, K.G. Selivanov and V.L. Telegdi,[Usp. Fiz. Nauk **169** 1141 (1999)], Phys. Usp. **42** 1045 (1999).

[21] Y.T. Hay *et al* Phys. Rev. Lett. **4** 165 (1960).

[22] J.H. Field, 'The physics of space and time III: Classification of space-time experiments and the twin paradox', arXiv pre-print: http://xxx.lanl.gov/abs/0806.3671v2. Cited 7 Nov 2011.

[23] J.H. Field, 'Primary and reciprocal space-time experiments, relativistic reciprocity relations and Einstein's train-embankment thought experiment', arXiv pre-print: http://xxx.lanl.gov/abs/0807.0158v2. Cited 11 Nov 2011.

[24] J.C. Hafele and R.E. Keating, Science **177** 166-168, 168-170 (1972).

[25] R.P. Feynman, R.B. Leighton and M. Sands, 'Lectures on Physics' Volume I (Addison-Wesley, Reading, Massachusetts, 1964) Section 15-4.

[26] Ref. [25], Section 15-5.

[27] N. Ashby, 'Relativity and the global positioning system', Physics Today, May 2002, pp.41-47.

[28] D.W. Allan, M.A. Weiss and N. Ashby, Science **228** 64-70 (1985).

[29] P Wolf and G. Petit, Phys. Rev. **A56** 4405 (1997).

[30] K. Schwartzschild, Sitzungberichte Prüssiche Akademie der Wissenschaften p.198 (1916).

[31] S. Weinberg, 'Gravitation and Cosmology, Principles and Applications of the General Theory of Relativity', (John Wiley, New York, 1972) Ch 8 Section 2.

[32] I. Shapiro, Phys. Rev. Lett. **13** 789 (1964).

[33] Ed. K. Elsener, 'The CERN Neutrino beam to Gran Sasso (Conceptual Technical Design)', CERN 98-02, INFN/AE-98/05.

[34] OPERA Collaboration, R. Acquafrdda *et al* JINST **4** P04018 (2009).

[35] M.G. Kuhn, 'The influence of Earth rotation in neutrino speed measurements between CERN and OPERA detector', arXiv pre-print: http://xxx.lanl.gov/abs/1110.03920v2. Cited 20 Oct 2011.

[36] G. Colosimo *et al* 'Determination of the CNGS global geodesy', Opera public note 132 v2. Cited 10 Oct 2011; http://operaweb.lngs.infn.it/Opera/publicnotes/note132.pdf

[37] OPERA Collaboration, T. Adam *et al* 'Measurement of the neutrino velocity with the OPERA detector in the CNGS beam', arXiv pre-print: http://xxx.lanl.gov/abs/1109.4897v1. Cited 22 Sept 2011.