The Sagnac effect and transformations of relative velocities between inertial frames

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Abstract

The Sagnac effect is analysed in both Galilean and Special relativity within a space-time geometrical model previously developed by Langevin and Post. The effect arises because of the different velocities of different light signals relative to the interferometer. The appropriate relativistic relative velocity transformation formulas obtained differ from the velocity transformation formulas of conventional Special relativity, the latter actually predicting, as previously pointed out by Dufour and Prunier and, more recently, by Selleri and Klauber, that the Sagnac effect vanishes. The Michelson-Morley experiment is analysed using the same model and a non-vanishing fringe shift, albeit below the sensitivity of all such experiments performed to date, is predicted. The Sagnac effect for neutrinos of the CERN CNGS beam is also discussed. The Sagnac effect indicates that the ECI (Earth Centered Inertial) frame is a preferred one in which light signals have a speed close to c, in the vicinity of the Earth, as predicted by General relativity.

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Sagnac published the results of his rotating interferometer experiment in 1913 [1]. The principle of the experiment is shown in Fig. 1. Light from a source S is split into two beams by a half-silvered mirror HSM. With the aid of the corner mirrors M_1, M_2, M_3 the light beams return to HSM via clockwise (HSM M_1 M_2 M_3 HSM) or counter-clockwise (HSM M_3 M_2 M_1 HSM) routes where they are combined into a single beam which is observed at D. When the whole apparatus, including the light source and the detector (which in Sagnac's original experiment was a photographic plate) is rotated a fringe shift ΔZ is observed, corresponding, at lowest order in the angular velocity, to a phase difference between the counter-rotating beams of: $\Delta\phi = 2\pi\Delta Z = 8\pi\vec{\Omega}\cdot\vec{A}/(\lambda_0 c)$ where $\vec{\Omega}$ is the angular velocity vector, λ_0 is the vacuum wavelength of the light, $|\vec{A}|$ is the area enclosed by the circulating light beams and $\vec{A}$ is perpendicular to the plane of the interferometer. This phase shift formula, for the case when $\vec{A}$ is parallel to $\vec{\Omega}$ and the axis of rotation passes through the center of a square interferometer, is derived below, from considerations of space-time geometry, in both Galilean and Special relativity.

It is interesting to note that, although Einstein had declared the luminiferous aether to be 'superfluous' in 1905 [2], the title of Sagnac's paper was 'L'éther lumineux démontré par l'effect du vent relatif d'éther dans un interférometer en rotation uniform', or, in English: 'Demonstration of the existence of the luminiferous aether by an aether wind effect in a rotating interferometer'. It was to search for just such an 'aether wind' that the Michelson-Morley experiment [3] and its successors [4, 5, 6, 7, 8, 9] were performed, with negative

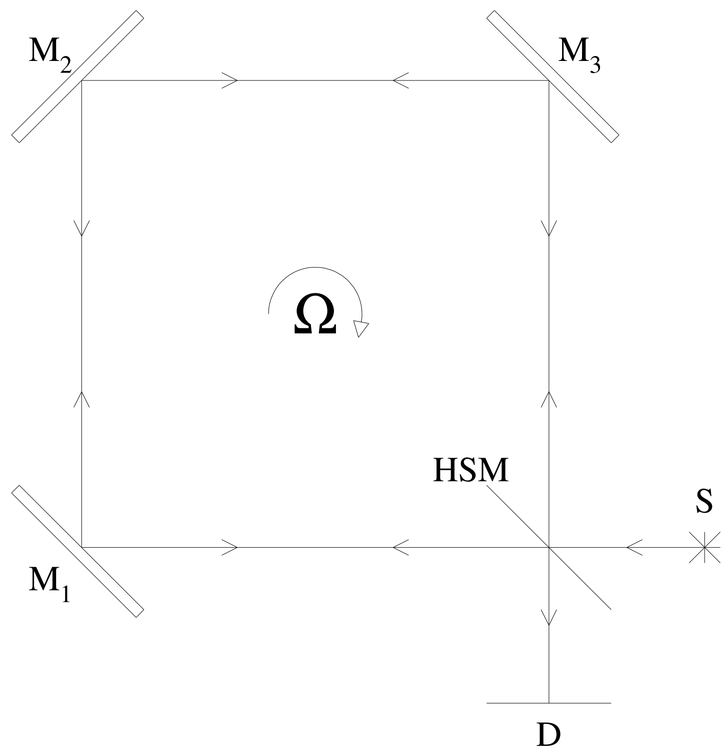


Figure 1: *A Sagnac interferometer. Light signals from a source S are split by the half-silvered-mirror HSM into two beams which follow clockwise (HSM M_1 M_2 M_3 HSM) or counter-clockwise (HSM M_3 M_2 M_1 HSM) paths, of equal length, back to HSM where they are recombined and detected at D. When the interferometer is rotated with angular velocity Ω, a phase shift develops between clockwise- and counter-clockwise-rotating beams due to different times-of-passage of the light signals. The latter result from different velocities of clockwise- and counter-clockwise-rotating light beams relative to the interferometer.*

results in almost all cases. As discussed below, this is not because an 'aether wind' does not exist, but because the corresponding phase shift is of order $(v/c)^2$ to be compared with order v/c for the observed phase shift in Sagnac's interferometer. Sagnac's experiment was repeated, with much improved precisison, by Pogany [10] and especially by Michelson and Gale [11] who used the effect to measure the speed of rotation of the Earth. More recently a related experiment —the ring laser— where counter-rotating laser beams have different characteristic frequencies when the device is rotated [12] was demonstrated and the Sagnac experiment itself was repeated using fibre optic light guides which enabled the development of highly sensitive fibre-optic gyroscopes [13, 14, 15] to detect rotation. Even more recently, it has been shown [16, 17], also by the use of fibre-optic interferometers, that relative translational motion also results in a phase shift due to the same space-time geometrical effect that underlies the original Sagnac experiment. The phase shift for translational motion is [16]: $\Delta\phi = 4\pi L\Delta v/(\lambda_0 c)$ where L is the length of the fibre optic path and Δv the change in the relative velocity of the light signals and the moving interferometer in the laboratory system. The present paper considers only the theory of the original rotating Sagnac experiment, however the essential underlying physics – different velocities of light signals relative to various elements of the interferometer— is the same for all Sagnac-type experiments whether in rotation or uniform translational motion.

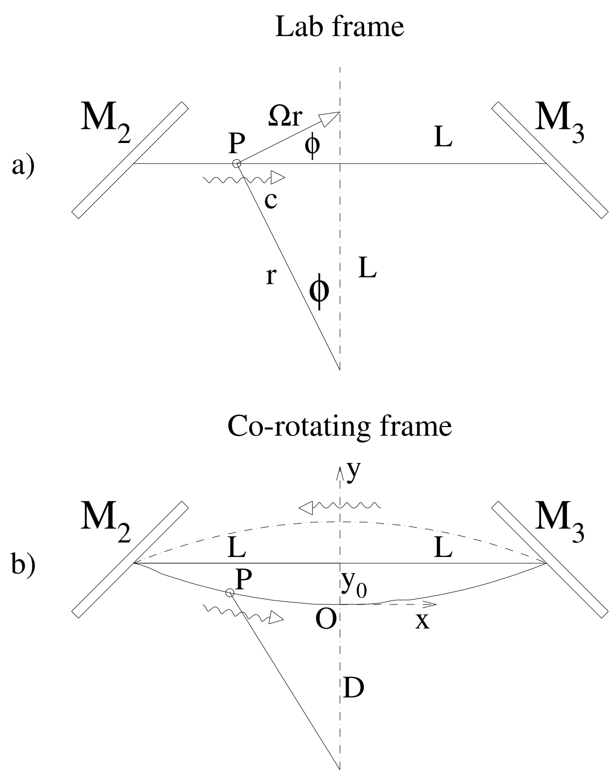


Figure 2: *Space-time geometry in Galilean relativity of the passage of a light signal between end mirrors M_2 and M_3 of the Sagnac interferometer shown in Fig. 1. a) in the laboratory frame; b) in the co-rotating frame of the interferometer. See text for discussion.*

The analysis will be first performed in the context of Galilean relativity, before considering the special relativistic analysis as previously done by Post [18], by suitable modification of a space-time geometrical calculation originally due to Langevin [19].

The fundamental space-time effect underlying the phase shift is a different transit time from beam-splitter to beam-splitter for clockwise- and counterclockwise-rotating beams, when the interferometer is rotating (see Fig. 1). In Fig. 2a a clockwise-moving photon with polar coordinates (r,Φ) in the laboratory frame is at position P in the rotating interferometer shown in Fig. 1. The velocity, c_r, of the photon, relative to P, parallel to the photon path in the laboratory frame, is, from the geometry of Fig. 2a:

$$c_r = c - \Omega r \cos\Phi = c - \Omega L. \tag{1}$$

where the length of the path of the light signal between successive mirrors is $2L$ when the interferometer is at rest. In Galilean relativity, neglecting the displacement of the mirrors in the laboratory frame, this is also the velocity of the photon, relative to the x-axis, in the co-rotating frame of the interferometer as shown in Fig. 2b. In this approximation (i.e. neglecting the rotation of the x-axis in Fig. 2b in the laboratory system during the passage of the photon from M_2 and M_3) the time dt_+ to cover an infinitesimal spatial interval dx including P is:

$$dt_+ = \frac{dx}{c_r} = \frac{r d\Phi}{\cos\Phi(c-\Omega L)} = \frac{L d\Phi}{\cos^2\Phi(c-\Omega L)} = \frac{L d(\tan\Phi)}{(c-\Omega L)}. \tag{2}$$

Integrating over the range: $-\pi/4 < \Phi < \pi/4$ gives

$$t_+ = \frac{L}{(c-\Omega L)}\int_{-1}^{1} d(\tan\Phi) = \frac{2L}{(c-\Omega L)}. \tag{3}$$

If T_+ (T_-) is the clockwise (counterclockwise) flight time of the photon from HSM to HSM, the 4-fold symmetry of the interferometer gives:

$$T_\pm = 4t_\pm = \frac{8L}{(c\mp\Omega L)}. \tag{4}$$

The phase shift due to rotation of the interferometer is then:

$$\begin{aligned}\Delta\phi_{\mathrm{GR}} &= 2\pi\nu(T_+ - T_-) = \frac{32\pi\nu\Omega L^2}{c^2(1-\beta(L)^2)} = \frac{8\pi\Omega A\gamma(L)^2}{\lambda_0 c} \\ &= \frac{8\pi\Omega A}{\lambda_0 c} + \mathrm{O}(\beta(L)^3) \qquad \text{(Galilean relativity)}\end{aligned} \tag{5}$$

where $\beta(L) \equiv \Omega L/c$, $\gamma(L) \equiv 1/\sqrt{1-\beta(L)^2}$ and $A = 4L^2$ is the area enclosed by the circulating light beams. The frequency ν here is that of the source as observed in the co-rotating frame. Since the distances between the source and the various elements of the interferometer are constant there is no classical Doppler effect

In the Appendix a space-time geometrical calculation in the laboratory frame, correct to order $\beta(L)^3$, taking into account the motion of the mirrors during photon transit is presented. It is found that

$$\Delta\phi_{\mathrm{GR}} = \frac{8\pi\Omega A}{\lambda_0 c}\left(1 + \frac{11\beta(L)^2}{24}\right) + \mathrm{O}(\beta(L)^5) \qquad \text{(Galilean relativity)} \tag{6}$$

in agreement, at first order in $\beta(L)$, with Eq. (5). It is also shown in the Appendix that, at order $\beta(L)$, the clockwise photon path, in the co-rotating frame of the interferomter, is the parabola:

$$y = \frac{\beta(L)x^2}{L} \tag{7}$$

in agreement with a previous calculation of Silberstein [20]. The paths, in the co-rotating frame of the interferometer, of clockwise (counterclockwise) moving photons, as shown in Fig. 2b, have a similar shape and are convex (concave) as viewed from the center of rotation.

The Sagnac interference phase is a consequence of different times of arrival of the counter-rotating signals back at the HSM. The appropriate time interval is therefore that recorded by a clock co-moving with the HSM. In the laboratory frame the HSM has a velocity of constant magnitude $\sqrt{2}\Omega L$, corresponding to the time dilation effect:

$$\Delta T = \frac{1}{\sqrt{1-2\beta(L)^2}}\Delta T' \tag{8}$$

so that

$$\Delta\phi_{\rm SR} = 2\pi\nu(T'_+ - T'_-) = \Delta\phi_{\rm GR}\sqrt{1-2\beta(L)^2}. \tag{9}$$

The frequency ν here is defined as in Eq. (5) in the case that the source and the HSM are at the same distance from the axis of rotation and so have the same velocity in the laboratory frame. If this is not the case, then the frequency of the light incident on the HSM will be shifted in frequency due to a differential time dilation effect [37, 38]. Combining (6) and (9):

$$\Delta\phi_{\rm SR} = \frac{8\pi\Omega A}{\lambda_0 c}\left[1 - \frac{13\beta(L)^2}{24}\right] + {\rm O}(\beta(L)^5). \quad \text{(Special relativity)} \tag{10}$$

Special relativity therefore contributes only an order $\beta(L)^2$ correction to the Sagnac phase difference as calculated in Galilean relativity.

As pointed out by Dufour and Prunier in 1937 [21], as well as later by Selleri [22, 23] and Klauber [24] the lowest order Galilean prediction of Eq. (5), as well as the relativistic prediction of Eq. (10), for $\Delta\phi$, is inconsistent with naive application of Einstein's second postulate of special relativity —that the speed of light is the same in all inertial frames. In the present problem, this postulate predicts that if the length of an element of the light path in the instantaneous comoving inertial frame of the fixed point P on the interferometer in Fig. 2b is $\delta s'$ and $\delta t'$ is the corresponding proper-time element in the frame then $\delta s'/\delta t' = c$, or since time dilation is a second order effect in $\beta(L)$:

$$\frac{\delta s'}{\delta t} = c + {\rm O}(\beta(L)^2).$$

Application of the second postulate then predicts that, considering a co-rotating light signal:

$$T_+ = \int dt = \frac{1}{c}\int ds' = \frac{s'}{c} + {\rm O}(\beta(L)^2). \tag{11}$$

From Eq. (7):

$$\int ds' = 4\int_{-L}^{L}\left(\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right)dx = 4\int_{-L}^{L}\left(\sqrt{1+\frac{4\beta(L)^2x^2}{L^2}}\right)dx$$
$$= 4\int_{-L}^{L}\left(1+\frac{2\beta(L)^2x^2}{L^2}\right)dx + \mathrm{O}(\beta(L)^4)$$
$$= 8L\left(1+\frac{2\beta(L)^2}{3}\right) + \mathrm{O}(\beta(L)^4). \qquad (12)$$

So that

$$T_+ = \frac{8L}{c}\left(1+\frac{2\beta(L)^2}{3}\right) + \mathrm{O}(\beta(L)^4). \qquad (13)$$

This result is in contradiction with Eq. (A20) of the Appendix which gives instead:

$$T_+ = \frac{8L}{c}\left(1+\beta(L)+\frac{\beta(L)^2}{2}+\frac{11\beta(L)^3}{24}\right) + \mathrm{O}(\beta(L)^4 \qquad (14)$$

Also replacing $\beta(L)$ by $-\beta(L)$ in Eq. (12) gives $T_- = T_+$ at all orders in $\beta(L)$, and so, as first pointed out by Dufour and Prunier [21], a vanishing Sagnac interference phase. Evidently this naive application of the second postulate of special relativity is inconsistent with the existence of the Sagnac effect.

It is interesting, in view of a comparison with the previously published work of Post [18], to also consider a circular geometry for the interferomenter (see Fig. 3) in which the relative velocities of the light signals and the interferometer are given by:

$$c_r^{\pm} = c \mp \Omega R \qquad (15)$$

where c_r^+ (c_r^-) are the velocites of clockwise (counterclockwise) rotating light signals, relative to an adjacent point on the interferometer, in the laboratory system, and R the radius of the circular light path. The times-of-passage of the light signals from beam-splitter to beam-splitter in the laboratory system for the counter-rotating signals are:

$$T_{\pm} = \frac{2\pi R}{c_r^{\pm}} = \frac{2\pi R}{c \mp \Omega R}. \qquad (16)$$

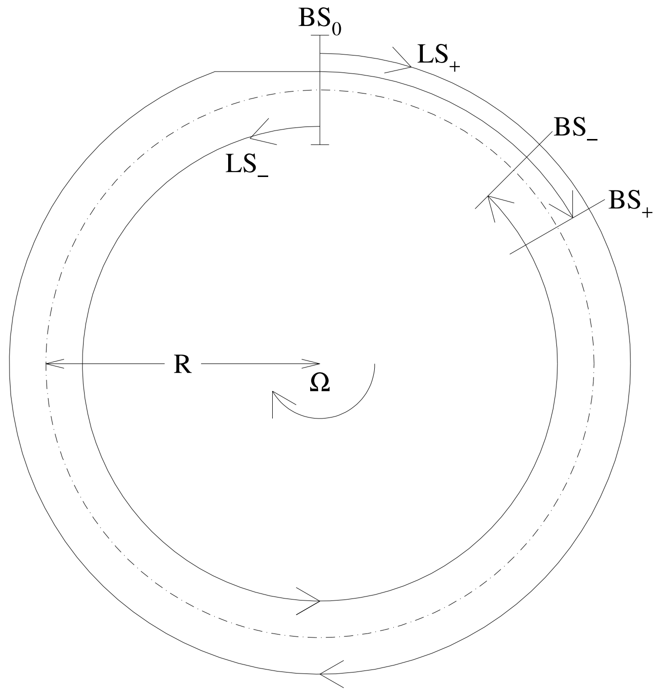
In Galilean relativity $T_{\pm} = T'_{\pm}$ where $T'_{\pm}$ are the times of passage in the co-rotating frame of the interferometer, so the corresponding Sagnac (S) phase shift is:

$$\Delta\phi^{\mathrm{S}}_{\mathrm{GR}} = 2\pi\nu(T_+ - T_-) = \frac{8\pi^2\nu R\beta(R)}{c(1-\beta(R)^2)} = \frac{8\pi\Omega A\gamma(R)^2}{\lambda_0 c}$$
$$= \frac{8\pi\Omega A}{\lambda_0 c}(1+\beta(R)^2) + \mathrm{O}(\beta(L)^5). \qquad \text{(Galilean relativity)} \qquad (17)$$

where $A = \pi R^2$.

The differential Lorentz transformations from the laboratory system into the instantaneous co-moving frame of the beam splitter BS in Fig. 3 are:

$$dS' = 0 = \gamma(R)(dS - c\beta(R)dt), \qquad (18)$$
$$dt' = \gamma(R)\left(dt - \frac{\beta(R)dS}{c}\right) \qquad (19)$$



Laboratory frame

Figure 3: *A circular Sagnac interferometer of radius R rotating with uniform angular velocity Ω in the clockwise direction. Co-rotating (LS_+) and counter-rotating (LS_-) light signals depart simultaneously from a beam splitter (BS) when it is positioned at BS_0. The signals LS_- (LS_+) arrive back at BS when it is in the laboratory frame positions BS_- (BS_+). In the laboratory frame both light signals move with speed c. The different arrival times result from different laboratory frame path lengths followed by the signals. Fot clarity the paths of the light signals, that have equal radii, are shown with small radial displacements.*

where dS is an element of the path in the laboratory frame of a point on the beam splitter at the same radial position as the light signals and dS' vanishes, since BS is, by definition, at rest in its co-moving frame. Combining (18) and (19) (eliminating dS between the two equations) yields the differential time dilation relation:

$$dt = \gamma(R)dt' \tag{20}$$

which on integration gives:

$$T'_\pm = \frac{T_\pm}{\gamma(R)} \tag{21}$$

so that the phase shift becomes:

$$\Delta\phi^{\rm S}_{\rm SR} = \frac{8\pi\Omega A\gamma(R)}{\lambda_0 c} = \frac{8\pi\Omega A}{\lambda_0 c}\left[1+\frac{\beta(R)^2}{2}\right] + {\rm O}(\beta(R)^5). \quad \text{(Special relativity)} \tag{22}$$

In view of the time dilation relations (21) the relative velocities of the light signals and the interferometer are not the same in the laboratory and co-rotating systems in the special relativistic case:

$$T'_\pm \equiv \frac{2\pi R}{(c_r^\pm)'} = \frac{T_\pm}{\gamma(R)} = \frac{2\pi R}{\gamma(R)[c \mp \Omega R]} = \frac{2\pi R}{\gamma(R) c_r^\pm} \tag{23}$$

These formulas for signal flight times in the co-rotating frame have been previously given by Tartaglia [25]. Eq. (23) shows that the relative velocities of the light signals and the interferometer transform between the laboratory and co-rotating frames as

$$(c_r^\pm)' = \gamma(R) c_r^\pm = \gamma(R)[c \mp \Omega R]. \tag{24}$$

in agreement with previous work by Klauber [24]. The corresponding formula for angular velocities and clockwise-rotating signals was derived[a] by Post [18]:

$$\omega' = \gamma(R)(\omega - \Omega) \tag{25}$$

where $\omega' \equiv (c_r^+)'/R$ and $\omega \equiv c/R$.

At order $\beta(R)$ the above analysis of a circular Sagnac interferometer is the same as that of Selleri [22, 23]. In particular Selleri gave [23] in both Galilean and (correctly applied) Special Relativity, the relation, that follows from Eqs. (24):

$$\frac{(c_r^+)'}{(c_r^-)'} = \frac{c - \Omega R}{c + \Omega R}.$$

The 'inertial transformations' employed by Selleri differ from the Lorentz transformation equations (18) and (19) used above only by invoking a spurious 'length contraction' effect arising from misinterpretation of the space transformation equation which is formally identical to Eq. (18).

[a] Post actually obtained, by consideration of the geometry of his Fig. 8, (which is similar to Fig. 3 of the present paper), by taking into account the time dilation effect in modifying the original Galilean calculation of Langevin [19], the formula $d\phi = d\phi' + \gamma(R)\Omega dt'$. This is Eq. (24) of [18]. From this follows: $d\phi/dt' = d\phi'/dt' + \gamma(R)\Omega$. Time dilation gives $d\phi/dt' = \gamma(R) d\phi/dt$ so that $\omega' = \gamma(R)(\omega - \Omega)$ where $\omega \equiv d\phi/dt$ and $\omega' \equiv d\phi'/dt'$.

The relative velocity transformation formula (24) differs markedly from the relativistic parallel velocity addition relation (RPVAR) [2] which gives:

$$(c^{\pm})' \equiv \frac{c \mp \Omega R}{1 \mp \frac{c(\Omega R)}{c^2}} = c\frac{1 \mp \frac{\Omega R}{c}}{1 \mp \frac{\Omega R}{c}} = c. \qquad (26)$$

The light signals are therefore predicted by the RPVAR to have (consistent with a naive application of the second postulate of Special relativity) the same velocity in the instantantaneous comoving inertial frame of a fixed point of the interferometer on the light path, as they do in the laboratory frame. In consequence $T'_{+} = T'_{-} = 2\pi R/c$ so that the Sagnac effect vanishes. It is clear from (26) that the usual relativistic velocity addition formulas are not applicable to the space-time analysis of a Sagnac interferometer.

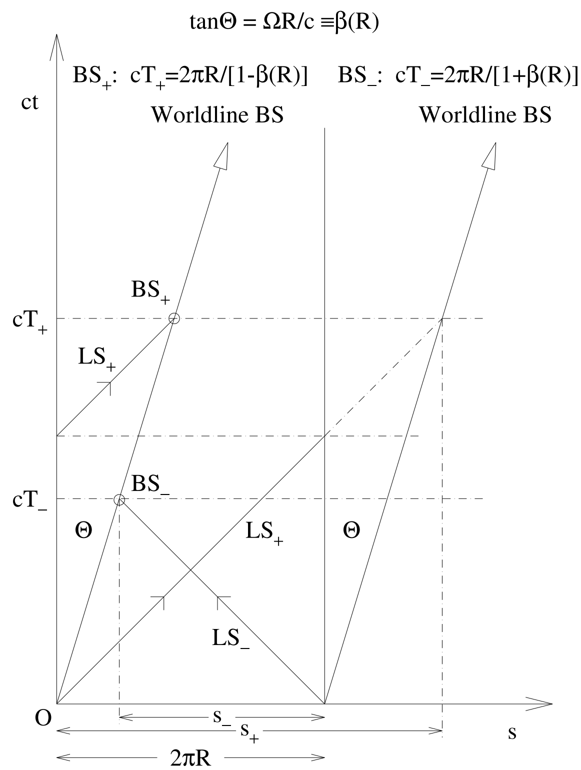


Figure 4: *Worldlines in the laboratory frame of the beam splitter (BS) and co-rotating (LS_{+}) and counter-rotating (LS_{-}) light signals in the circular Sagnac Interferometer shown in Fig. 3. See text for discussion.*

The space-time geometry of the passages of the light signals BS_{+} and BS_{-} of Fig. 3 is shown in Fig. 4 in the laboratory system and Fig. 5 in the co-rotating frame of the interferometer. The abscissas in Fig. 4(5) are the clockwise-oriented path lengths s (s') separating the light signals from the initial position of BS. Due to the azimuthal symmetry of the circular paths of the light signals:

$$x = x - 2\pi R \times INT\left(\frac{x}{2\pi R}\right) \quad (x = s, s') \qquad (27)$$

where $INT(X)$ is the integer part of the real number X. Inspection of Fig. 5 shows clearly the incompatiblity of the existence of the Sagnac effect with the the second postulate of special relativity. If the light signals propagate at speed c relative to BS they follow the 'lightcone' worldlines LC_+ or LC_- and it is predicted that:

$$T'_+ = T'_- = \frac{2\pi R}{c}. \qquad (28)$$

Thus the light signals are predicted to arrive simultaneously at BS in its co-moving frame. This is in violation of the 'zeroth theorem' of spacetime physics as enunciated by Langevin [26, 27] and more recently invoked by Mermin [28]. A triple space-time coincidence —a common event on the world lines of both light signals and BS— is predicted to occur in the co-moving frame of BS. All observers must agree that such an event exists, whereas inspection of Fig. 4 shows that there is no such event in the laboratory frame. Alternatively it may be noted that the prediction $\Delta T' \equiv T'_+ - T'_- = 0$ corresponding to the world lines LC_+ and LC_- is inconsistent with the time dilation relations (21) and the non-vanishing value of $T_+ - T_-$ given by Eq. (16). The latter is a consequence only of space-time geometry in the laboratory frame and is valid in both Galilean and Special relativity.

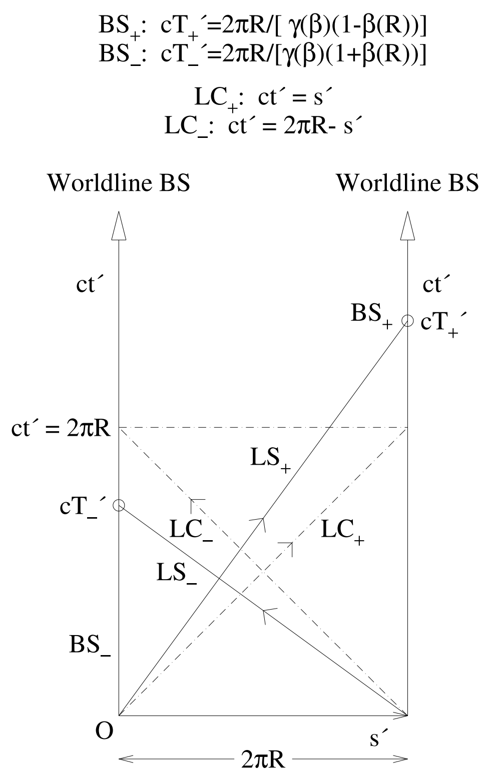


Figure 5: *Worldlines in the co-rotating frame of the circular Sagnac Interferometer shown in Fig. 3: Beam splitter (BS), co-rotating light signal (LS_+) and counter-rotating light signal (LS_-). Also shown are the world lines of the light signals, LC_+ and LC_- as predicted by application of the RPVAR Eq. (26), See text for discussion.*

A common error in the literature, due originally to Langevin [29] and widely propagated due to its effective inclusion in the discussion of the Sagnac effect in a textbook by Landau and Lifshitz [30], is the derivation of a formula for $T_+ - T_-$ by misinterpretation of the time Lorentz transformation between the instantaneous comoving inertial frame of a fixed point on the interferometer and the laboratory frame:

$$t = \gamma(R)\left(t' + \frac{\beta(R)S'}{c}\right) \tag{29}$$

which is the inverse of the integrated version of Eq. (19) above. Assuming the speed of the light signals in the co-moving frame of LS is c, then $t' = T'_+ = T'_- = C'/c$ where C' is the length of the light paths in this frame. Then setting $S' = S'_+ = C'$ or $S' = S'_- = -C'$ in (29) gives:

$$\tilde{T}_+ \equiv \gamma(R)\frac{C'}{c}(1+\beta(R)), \tag{30}$$

$$\tilde{T}_- \equiv \gamma(R)\frac{C'}{c}(1-\beta(R)). \tag{31}$$

So that

$$\Delta\tilde{T} = \tilde{T}_+ - \tilde{T}_- = \frac{2\gamma(R)\beta(R)C'}{c}. \tag{32}$$

Further assuming length contraction in the laboratory frame of the circumference of the circular light paths, as suggested by Ehrenfrest [31][b]: $C \equiv 2\pi R = C'/\gamma(R)$, (32) gives:

$$\Delta\tilde{T} = \frac{4\pi R\gamma(R)^2\beta(R)}{c} \tag{33}$$

in agreement with the prediction of (16) above:

$$\Delta T \equiv T_+ - T_- = \frac{4\pi R\beta(R)}{c(1-\beta(R)^2)}. \tag{34}$$

The calculation above yielding $\Delta\tilde{T}$ is claimed to show that the Sagnac effect is purely relativistic, being an example of the 'relativity of simultaneity' effect of special relativity. This calculation misinterprets the time-interval Lorentz transformation (29). The quantities t, t' and S' in this equation are time or space intervals *along the world line of the beam splitter* not the worldlines of the light signals! Since t' and S' are defined in the proper frame of the latter where (by definition) it is at rest, $S' = \int dS' = 0$, since, from Eq. (18), $dS' = 0$. Correctly interpreted therefore, (29) reduces to

$$t = \gamma(R)t' \tag{35}$$

equivalent to the time dilation relations (21). There is no 'relativity of simultaneity' and the velocity of light must be anisotropic in the proper frame of the beam splitter, in contradiction to the second postulate of special relativity. The motivation of the choice of non-zero values of S'_+ and S'_- with different signs, in the derivation of Eq. (33), arises from their false identification with the paths $s'_\pm$ of the counter-rotating light signals in the proper frame of the beam splitter (see Fig. 5) rather than the null path length of the beam splitter itself in this frame. Actually there is only one one conceptual 'clock' in the analysis of the Sagnac effect, that is situated on the beam splitter, so that the

[b]Note, in contrast, that Einstein gave arguments [32, 33] according to which the ratio of the circumference of a rotating disc to its radius, as viewed from an inertial frame at rest relative to its center, should be *greater* than 2π.

putative 'relativity of simultaneity' effect for two clocks at different spatial locations, can have no possible relevance to the problem. Also since the Sagnac effect corresponds to a *time interval* registered by a single (conceptual) clock, then, contrary to a vast scientific literature[c], any considerations of the relative synchronisation of spatially separated clocks [35], is irrelevant to a correct understanding of the effect. See Ref. [36] for a critique of the conventional derivation of 'relativity of simultaneity' from the space-time Lorentz transformations.

Writing the RPVAR in terms of scaled velocities $\beta_v \equiv v/c$:

$$\beta_{u'} = \frac{\beta_u - \beta_v}{1 - \beta_u \beta_v} \tag{36}$$

it is straightforward to show that this equation is mathematically equivalent to [d] either of the formulas:

$$\gamma_{u'} = \gamma_u \gamma_v (1 - \beta_u \beta_v), \tag{37}$$

$$\gamma_{u'} \beta_{u'} = \gamma_u \gamma_v (\beta_u - \beta_v) \tag{38}$$

where $\gamma_v \equiv 1/\sqrt{1-\beta_v^2}$, which, in turn, are, respectively, equivalent to the transformation relations of relativistic energy: $E = \gamma_v m c^2$, and momentum: $p = \gamma_v m v$. Thus one correct physical interpretation of the RPVAR is to be found in relativistic kinematics rather than in space time geometry. For further discussion of this important point see Refs. [39, 40, 41]. The formula (37) also gives the transformation of the time dilation factor γ between different inertial frames, as may be exemplified by its application to the Hafele-Keating experiment [42].

The Michelson-Morley (MM) experiment will now be analysed in the same manner as the Sagnac interferometers discussed above, i.e. it will be assumed that the speed of light has the value c in the laboratory frame and the formula (24) will be used to find the relative velocity of the light signals in the rest frame of the interferometer, which is an inertial frame, rather than the uniformly rotating one of a Sagnac interferometer. The appropriate 'laboratory frame' for experiments performed on the surface of the Earth will be discussed below. The analysis of light signals in the transverse arm of a Michelson interferometer is familar from elementary derivations of the time dilation relation [43] — the velocity of the light signals is equal to c, both in the laboratory frame and in the rest frame of the interferometer— so that, if the length of each arm is D, the time-of-passage in the transverse arm in the interferometer rest frame is $T'_{\rm T} = 2D/c$. If v is the velocity of interferometer in the laboratory system, then on making the replacement $\Omega R \to v$ in (24), the time-of-passage in the rest frame of the interferometer of the light signal in the longitudinal arm is:

$$T'_{\rm L} = \frac{D}{c\gamma_v}\left[\frac{1}{1-\beta_v} + \frac{1}{1+\beta_v}\right] = \frac{2D\gamma_v}{c}. \tag{39}$$

If the interferometer is rotated through 90° around a vertical axis the longitudinal and transverse arms are exchanged resulting in a phase shift proportional to twice the time difference $T'_{\rm L} - T'_{\rm T}$:

$$\Delta\phi_{\rm SR}^{\rm MM} = 2[2\pi\nu(T'_{\rm L} - T'_{\rm T})] = \frac{8\pi D}{\lambda_0}(\gamma_v - 1) = \frac{4\pi D}{\lambda_0}\beta_v^2 + {\rm O}(\beta_v^4). \quad \text{(Special relativity)} \tag{40}$$

[c] See, for example, Chapters 1,2,5,7,9,10 of the book 'Relativity in Rotating Frames' [34] and references therein.

[d] That is, by postulating any one of Eqs. (48), (49) and (50) the remaining two may be obtained by purely algebraic manipulation.

In Galilean relativity the phase shift is, at $O(\beta_v^2)$, a factor of two larger. For comparison the phase shift in the Sagnac interferometer of Fig. 1 may be written as:

$$\Delta\phi_{\rm SR}^{\rm S} = \frac{32\pi L}{\lambda_0}\beta(L) + O(\beta(L)^3). \tag{41}$$

The β_v^2 dependence of $\Delta\phi_{\rm SR}^{\rm MM}$ as compared to the $\beta(L)$ dependence of $\Delta\phi_{\rm SR}^{\rm S}$ explains why the Sagnac experiment successfully detected an 'aether wind' on the surface of the Earth while the MM experiment and later improved versions operating on the same principle failed to do so. In the Michelson-Gale Sagnac experiment situated at latitude 41°46′N the value of $\beta(L)$ for the East-West pointing arm of the interferometer due to the rotation of the Earth was $(0.34 \text{ km/s})/c = 1.1 \times 10^{-6}$. The corresponding Sagnac phase shift was 0.23 of a fringe width. Placing a Michelson interferometer with a similar light source and dimensions $2L \simeq D = 0.5$ km at the same latitude as the Michelson-Gale experiment Eqs. (40) and (41) predict, for the ratio of phase shifts:

$$\frac{\Delta\phi_{\rm SR}^{\rm MM}}{\Delta\phi_{\rm SR}^{\rm S}} = \frac{\beta(L)}{4} = 2.8 \times 10^{-7} \tag{42}$$

corresponding to a phase shift of 6.3×10^{-8} of a fringe in the Michelson interferometer.

In interpreting the results of the MM experiment and its successors it was usually assumed that the 'aether' was at rest relative to the Solar System which corresponds to a value of β_v in (40) equal the the speed of rotation of the Earth around the Sun of 29.8 km/s so that $\beta_v \simeq 10^{-4}$. This gives a phase shift in a Michelson interferometer 10^4 times larger than a value of β_v corresponding to the rotation of the Earth about its polar axis. The upper limit: $\beta_v \simeq 10^{-5}$ ($v \simeq 10$ km/s) obtained by the Kennedy-Thorndike experiment [7], which has a sensitivity of about 10^{-5} of an interference fringe width, was still some 30 times larger than the velocity of the surface of the Earth in the Michelson-Gale experiment. At least another two orders of magnitude improvement in the sensitivity of a Michelson interferometer would therefore be needed to detect the speed of the 'aether wind' generated by the rotation of the Earth.

The space-time geometry for the passage of a light signal in the longitudinal arm of a Michelson interferometer between the half-silvered-mirror HSM and the end mirror of the arm, $\rm M_L$ is shown in Fig. 6. The light signal travels at speed c in the laboratory frame while the velocity v of the interferometer in the West-to-East direction, is due to the rotation of the Earth, i.e. the interferometer is at rest on the surface of the Earth. The light signal leaves HSM at laboratory time $t = 0$ and reaches $\rm M_L$ when $t = t_{\rm L}$. The geometry of Fig. 6b gives:

$$ct_{\rm L} = vt_{\rm L} + D(\text{lab}) \tag{43}$$

where D(lab) is the separation of HSM and $\rm M_L$ in the laboratory frame. The speed of the light signal in the rest frame of the interferometer is given by Eq. (24) as:

$$(c_r^+)' = \gamma_v(c - v) \tag{44}$$

so that the time-of-passage of the light signal in the rest frame of the interferometer, $t'_{\rm L}$, is

$$t'_{\rm L} = \frac{D}{(c_r^+)'} = \frac{D}{\gamma_v(c-v)} = \frac{Dt_{\rm L}}{\gamma_v D(\text{lab})} \tag{45}$$

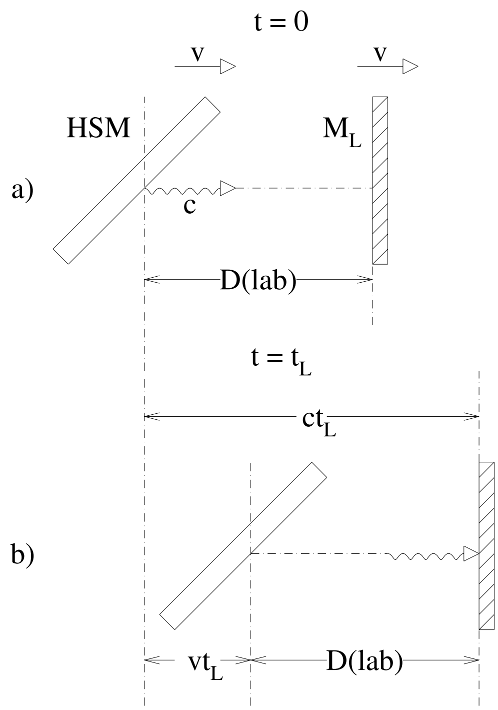


Figure 6: *Space-time geometry of the passage of a light signal in the longitudinal arm of a Michelson interferometer. The latter is at rest on the surface of the Earth with the arm directed in the West-to-East direction. The laboratory frame is the ECI frame and the velocity v of the interferometer is due to the rotation of the Earth. See text for discussion.*

where, in the last member, Eq. (43) is used, after transposition, to eliminate $c - v$. Combining the time dilation relation

$$t_{\rm L} = \gamma_v t'_{\rm L} \tag{46}$$

with (45) then shows that

$$D({\rm lab}) = D. \tag{47}$$

There is no 'length contraction' effect.

In the conventional interpretation of the MM experiment the failure to observe any phase shift between signals in the longitudinal and transverse arms is assumed to imply that $T'_{\rm L} = T'_{\rm T}$. The space-time geometry of the laboratory frame gives a time-of-passage of the light signal from HSM to $\rm M_L$ and back, in this frame, of:

$$T_{\rm L} = D({\rm lab})\left[\frac{1}{c-v} + \frac{1}{c+v}\right] = \frac{2D({\rm lab})}{c(1-\beta_v^2)}. \tag{48}$$

Now

$$T'_{\rm T} = \frac{2D}{c} \tag{49}$$

and time dilation gives:

$$T_{\rm L} = \gamma_v T'_{\rm L}. \tag{50}$$

Combining (48), (49) and (50), on the assumption $T'_{\rm L} = T'_{\rm T}$ (no phase shift), gives

$$T_{\rm L} = \gamma_v T'_{\rm L} = \gamma_v T'_{\rm T} = \frac{2\gamma_v D}{c} = \frac{2D({\rm lab})}{c(1-\beta_v^2)} \tag{51}$$

from which follows:

$$D({\rm lab}) = \frac{D}{\gamma_v} \quad (T'_{\rm L} = T'_{\rm T}). \tag{52}$$

This is the 'length contraction' effect [44] which explains a null result for the MM experiment in conventional Special relativity. As explained above, for consistency with the observed Sagnac effect, a non-vanishing phase shift must exist in a MM-type experiment, but, to date, no such experiment has had sufficient sensitivity to observe the phase shift. For further critical discussion of the putative special relativistic 'length contraction' and 'relativity of simultaneity' effects see [36, 39, 40, 41] and references therein.

As described in Refs. [45, 46] corrections for the Sagnac effect are routine in the operation of the Global Positioning System (GPS). The velocity of GPS microwave signals in the rest frame of a GPS receiver are calculated according to the Galilean formula (1) above. Similar corrections are applied in tests, using the GPS, of the isotropy of the speed of light [47]. In this case, as also in the Michelson-Gale experiment, the 'laboratory frame', in which the speed of light is assumed [45, 46] or measured [47] to be c, is the Earth-Centered-Inertial (ECI) frame which is the co-moving inertial frame of the centroid of the Earth with axes pointing to fixed directions on the celestial sphere. It is in this frame that the Earth's gravitational field is given by the Schwartschild metric [48, 49] and which effectively contains the 'aether', relative to which, 'winds' were observed by Sagnac, and Michelson and Gale. It is indeed a prediction of General relativity that, in just this frame, the speed of light is (very nearly) equal to c. 'Very nearly' because of the Shapiro delay [50] of light signals crossing the Earth's gravitational field. For signals from the GPS satellites such delays are less than 200ps [45] and so give no perceptible effect in GPS operation.

For hypothetical in vacuo light signals circumnavigating the Earth at the Equator at constant distance R from the center of the Earth the velocity of light is given by the Schwartschild metric equation:

$$0 = (d\tau)^2 = \left(1 + \frac{2\phi_{\rm E}}{c^2}\right)(dt)^2 - \frac{R^2 d\phi^2}{c^2} \tag{53}$$

where $\phi_{\rm E} = -GM_{\rm E}/R$ is the gravitational potential due to the Earth. Then the speed of the light signals in the ECI frame is;

$$c_{\rm E} \equiv \frac{R d\phi}{dt} = \left(1 + \frac{\phi_{\rm E}}{c^2}\right) c + {\rm O}[(\phi_{\rm E}/c^2)^2] \tag{54}$$

The values of the mass of the Earth, $M_{\rm E}$, and its equatorial radius, R, give $\phi_{\rm E}/(c^2) = -0.694 \times 10^{-9}$ so that

$$\frac{c - c_{\rm E}}{c} = 6.94 \times 10^{-10}$$

The 'Shapiro delay' for such a light signal is then about 90ps for a round trip time of $2\pi R/c_{\rm E} = 134$ms.

The existence of different 'effective aethers' around the Earth and the Sun in order to explain experimental data on the propagation of microwaves near to the surface of the Earth [45, 46, 51] and the Shapiro radar echo delay experiments for microwave signals passing close to the Sun [50] was proposed by Su [52, 53] in the context of a classical electromagnetic wave theory distinct from that given by Special relativity. However, as pointed out above, the existence of such 'effective aethers' is a necessary consequence of General relativity, so that no new classical theory of the type proposed by Su is required.

The Sagnac effect for neutrinos of the CERN CNGS beam [54] as detected in OPERA [55] in the Gran Sasso Laboratory has recently been considered [56]. Neutrinos, with energies around 17 GeV, from decays of charged pions or kaons are directed in a roughly South-Easterly direction through the crust of the Earth and are detected after a flight distance of about 730 km in the underground detector OPERA. As for photons in the Sagnac and Michelson-Gale experiments, the neutrinos are expected to have speed c in the ECI frame. During the 2.4 ms time-of-flight of the neutrinos the OPERA detector moves a distance 0.835 m [56] in an Easterly direction due to the rotation of the Earth. This increases the time-of-flight of the neutrinos by 2.2 ns [56]. This implies, in turn, that that the neutrinos have an average speed, in the co-moving inertial frame of OPERA (in which the CERN-OPERA separation is constant), that is less than c by the fraction 9.2×10^{-7}. Notice that if the CERN neutrino beam were instead directed in a South-Westerly direction the measured speed of the neutrinos would be, by a similar fraction, *greater than* c, so that, when the Sagnac effect is taken into account, speeds of particles relative to detectors are not limited to be less than or equal to c. Making use of detailed survey information on the positions of the neutrino source and the OPERA detector [57] the angle, α, between the neutrino beam direction and the direction of motion of OPERA in the ECI frame is found to be $\alpha = 37.8^\circ$.

According to the conventional velocity transformation formulas of special relativity [2] the velocity components of the neutrinos, parallel to ($v_{\parallel}$), and perpendicular to ($v_{\perp}$), the direction of motion of OPERA, in the co-moving inertial frame of the latter, when they are assumed to have speed c in ECI frame, are:

$$v_{\parallel} = c\left[\frac{\cos\alpha - \beta_{\rm O}}{1-\beta_{\rm O}\cos\alpha}\right] \tag{55}$$

$$v_{\perp} = c\left[\frac{\sin\alpha}{\gamma_{\rm O}(1-\beta_{\rm O}\cos\alpha)}\right] \tag{56}$$

where $\beta_{\rm O} \equiv v_{\rm O}/c$, $\gamma_{\rm O} \equiv 1/\sqrt{1-\beta_{\rm O}^2}$ and $v_{\rm O} = 323$ m/s is the speed of OPERA in the ECI frame. The speed, v, of the neutrinos in the OPERA frame is then given by:

$$\begin{aligned} v^2 = v_{\parallel}^2 + v_{\perp}^2 &= c^2\left[\frac{(\gamma_{\rm O}^2-1)\cos^2\alpha - 2\beta_{\rm O}\gamma_{\rm O}^2\cos\alpha + \gamma_{\rm O}^2\beta_{\rm O}^2+1}{\gamma_{\rm O}^2(1-\beta_{\rm O}\cos\alpha)^2}\right] \\ &= c^2\left[\frac{\gamma_{\rm O}^2\beta_{\rm O}^2\cos^2\alpha - 2\beta_{\rm O}\gamma_{\rm O}^2\cos\alpha + \gamma_{\rm O}^2}{\gamma_{\rm O}^2(1-\beta_{\rm O}\cos\alpha)^2}\right] \\ &= c^2 \end{aligned} \tag{57}$$

(where the identity $\gamma_{\rm O}^2 \equiv \gamma_{\rm O}^2\beta_{\rm O}^2 + 1$ has been used) so that the Sagnac effect vanishes, in contradiction with the prediction of Ref. [56].

The current OPERA measurement [58] of the neutrino time-of-flight gives, at 90% confidence, a value of v such that:

$$-1.8\times 10^{-6} < (v-c)/c < 3.0\times 10^{-6}$$

In order for the velocity measurement to be sensitive to the Sagnac effect: $\Delta c/c = -9.2\times 10^{-7}$, at least an order of magnitude reduction in the systematic uncertainty of the time-of-flight measurement (currently $\simeq$ 3ns) is required. In contrast, the uncertainty of 20 cm in the flight distance given in [57] is about a factor of four less than the displacement due to the Sagnac effect. Further discussion of measurements of the Sagnac effect in existing and planned terrestrial long-baseline neutrino beams may be found in Ref. [59]

The final conclusions are that the ECI frame constitutes a physically-preferred reference system for light signals or neutrinos in the vicinity of the Earth and that the Sagnac effect is not correctly described, either for light signals (photons) or neutrinos by the velocity transformation formulas of conventional special relativity. There is clearly an important mismatch between what is known and applied by engineers in the GPS system [45, 51], and in practical applications of the Sagnac effect (for example fiber-optic gyroscopes [13, 14, 15]) and the content of the scientific literature and text books on Special relativity theory, that needs to be rectified.

Appendix

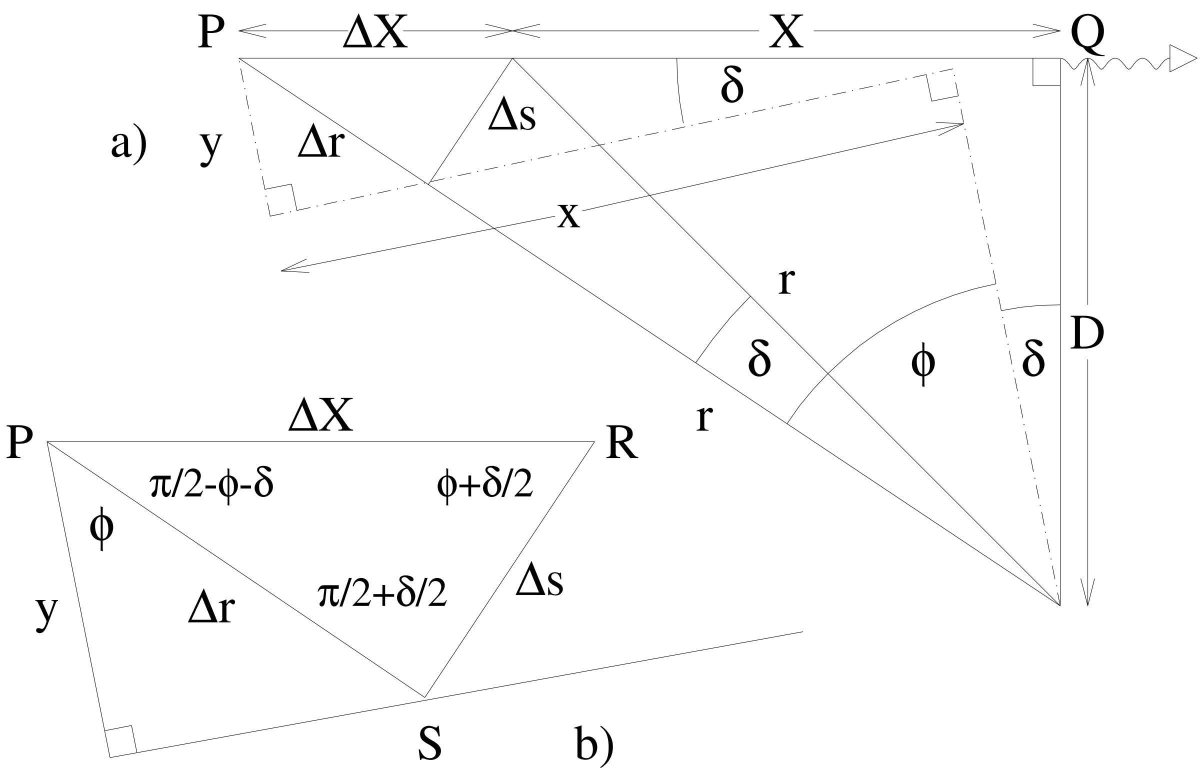


Figure 7: *a) Laboratory frame configuration of the upper arm of the square Sagnac interferometer shown in Fig. 1. The axes of the Cartesian coordinates x and y are fixed in the interferometer frame as shown in Fig. 2b. The light signal at point Q, midway between the mirrors M_2 and M_3, is at the point P at an earlier time, corresponding to rotation in the laboratory frame by an angle δ of the x and y axes. b) Angles of the triangle PRS in terms of the angles δ and ϕ defined in a).*

In Fig. 7. is shown the laboratory frame configuration when a co-rotating light signal is at the point Q, midway between the mirrors M_2 and M_3 of Fig. 1 or 2. In the laboratory frame the signal follows a straight line path between the reflections at the mirrors. Assuming that the source S in Fig. 1 is close to the HSM the light signals impact the mirrors at a mean distance $\sqrt{2}L$ from the axis of rotation, which is at the middle of the square light path in Fig. 1. The point P on the light path, corresponding to the position of the light signal at an earlier time, is specified by Cartesian coordinates x, y fixed in the co-rotating frame of the interferometer, as shown in Fig. 2b. The x-coordinate origin is midway between the mirrors M_2 and M_3. When the light signal is at P, the x-axis is rotated by an angle δ relative to the X-axis, fixed in the laboratory frame, that is parallel to the path of the light signal in this frame. The geometry of Fig. 7 gives the following relations:

$$\begin{aligned}
\Delta t &= \frac{X+\Delta X}{c} = \frac{\delta}{\Omega}, && \text{(A.1)}\\
\frac{X}{D} &= \tan\phi = \frac{x}{D+y}, && \text{(A.2)}\\
y &= \Delta r\cos\phi, && \text{(A.3)}\\
\Delta s &= 2r\cos(\pi/2-\delta/2) = \frac{2D\cos(\pi/2-\delta/2)}{\cos\phi} && \text{(A.4)}
\end{aligned}$$

where Δt is the time-of-passage in the laboratory frame of the light signal between P and Q. Application of the Sine Rule to the triangle PRS in Fig. 4b gives:

$$\frac{\Delta r}{\sin(\phi+\delta/2)} = \frac{\Delta X}{\sin(\pi/2+\delta/2)} = \frac{\Delta s}{\sin(\pi/2-\phi-\delta)}. \tag{A.5}$$

Combining (A.1), (A.2), (A.3) and the last member of (A.5) gives

$$\begin{aligned}
\frac{\delta}{\beta_D} &= \tan\phi + \frac{2\sin(\pi/2-\delta/2)\sin(\pi/2+\delta/2)}{\cos\phi\sin(\pi/2-\phi-\delta)}\\
&= \tan\phi + \frac{\delta}{\cos^2\phi} + \frac{\delta^2\tan\phi}{\cos^2\phi} + \mathrm{O}(\delta^3) && \text{(A.6)}
\end{aligned}$$

where $\beta_D \equiv D\Omega/c$. Solving (A.6) for δ, by iteration, to successive orders in β_D, gives:

$$\begin{aligned}
\delta_1 &= \beta_D\tan\phi + \mathrm{O}(\beta_D^2), && \text{(A.7)}\\
\delta_2 &= \beta_D\tan\phi + \frac{\beta_D^2\tan\phi}{\cos^2\phi} + \mathrm{O}(\beta_D^3), && \text{(A.8)}\\
\delta_3 &= \beta_D\tan\phi + \frac{\beta_D^2\tan\phi}{\cos^2\phi} + \frac{\beta_D^3\tan\phi}{\cos^2\phi}(1+2\tan^2\phi) + \mathrm{O}(\beta_D^4). && \text{(A.9)}
\end{aligned}$$

Combining (A.3), (A.4) and (A.5) so as to eliminate Δr and Δs gives

$$\begin{aligned}
y &= \frac{2D\sin(\phi+\delta/2)\sin\delta/2}{\cos(\phi+\delta)}\\
&= D[\delta\tan\phi + \frac{\delta^2}{2}\sec^2\phi] + \mathrm{O}(\delta^3). && \text{(A.10)}
\end{aligned}$$

The path of the light signal in the co-rotating frame is then given by (A.2), (A.7) and (A.10) as

$$y_1 = \frac{\beta_D x^2}{D} + \mathrm{O}(\beta_D^2) \tag{A.11}$$

which, at order β_D, is a parabola. An iterative solution of (A.10) using (A.2) and (A.8) gives, up to second order in β_D:

$$y_2 = \frac{\beta_D x^2}{D}\left[1 - \frac{\beta_D}{2}\left(\frac{3x^2}{D^2} - 1\right)\right] + \mathrm{O}(\beta_D^3). \tag{A.12}$$

Associating P with the point of reflection of the light signal on the mirror M_2, then $\tan\phi = 1$ so that from (A.2) (see also Fig. 2):

$$x = x_0 = D + y_0 = L. \tag{A.13}$$

Setting $\phi = \pi/4$ in (A.9) gives for the corresponding value of the angle δ:

$$\delta_0 = \beta_D + 2\beta_D^2 + 4\beta_D^3 + \mathrm{O}(\beta_D^4) \tag{A.14}$$

while setting $\phi = \pi/4$ in the first member of (A.10) and retaining order δ^3 terms gives

$$y_0 = D\left(\delta_0 + \frac{3\delta_0^2}{2} + \frac{41\delta_0^3}{24}\right) + \mathrm{O}(\delta_0^4). \tag{A.15}$$

Combining (A.13)-(A.15) gives, after some algebraic manipulation, the relation between D and L in terms of β_D:

$$D = L\left(1 - \beta_D - \frac{5\beta_D^2}{2} - \frac{127\beta_D^3}{24}\right) + \mathrm{O}(\beta_D^4) \tag{A.16}$$

as well as the relation between β_D and $\beta_L \equiv (L\Omega)/c$:

$$\beta_D = \beta_L - \beta_L^2 - \frac{3\beta_L^3}{2} + \mathrm{O}(\beta_L^4). \tag{A.17}$$

The time-of-passage in the laboratory frame, T_+, of the co-rotating light signal from and back to the HSM is given by (A.1) as

$$T_+ = 8\frac{(D + \Delta X_0)}{c} \tag{A.18}$$

where

$$\begin{aligned} \Delta X_0 &= \left.\frac{D\sin\delta}{\cos\phi\cos(\phi+\delta)}\right|_{\phi=\frac{\pi}{4}} = \frac{2D\sin\delta_0}{\cos\delta_0 - \sin\delta_0} \\ &= 2D\left(\delta_0 + \delta_0^2 + \frac{4\delta_0^3}{3}\right) + \mathrm{O}(\delta_0^4). \end{aligned} \tag{A.19}$$

Combining (A.16)-(A.19) gives the final result for T_+ in terms of the dimension L of the interferometer and its angular velocity:

$$T_+ = \frac{8L}{c}\left(1 + \beta_L + \frac{\beta_L^2}{2} + \frac{11\beta_L^3}{24}\right) + \mathrm{O}(\beta_L^4). \tag{A.20}$$

Note that the time-of-passage at order β_L^3 is required for consistency, at this order, with the relativistic time dilation correction (see the main text). For counter-rotating signals the time-of-passage is given by setting β_L to $-\beta_L$ in (A.20):

$$T_- = \frac{8L}{c}\left(1 - \beta_L + \frac{\beta_L^2}{2} - \frac{11\beta_L^3}{24}\right) + \mathrm{O}(\beta_L^4). \tag{A.21}$$

The Sagnac phase shift in Galilean Relativity (GR) is therefore:

$$\Delta\phi_{\mathrm{GR}} = 2\pi\nu(T_+ - T_-) = \frac{8\pi\Omega A}{\lambda_0 c}\left(1 - \frac{11\beta_L^2}{24}\right) + \mathrm{O}(\beta_L^5). \tag{A.22}$$

This is Eq. (6) of the main text.

Using (A.16) and (A.17) to write (A.12) in terms of L and β_L leaves the form of the equation unchanged:

$$y_2 = \frac{\beta_L x^2}{L}\left[1 - \frac{\beta_L}{2}\left(\frac{3x^2}{L^2} - 1\right)\right] + \mathrm{O}(\beta_L^3) \tag{A.23}$$

so that

$$y_1 = \frac{\beta_L x^2}{L} + \mathrm{O}(\beta_L^2). \tag{A.24}$$

This is Eq. (7) of the main text.

References

[1] G. Sagnac, Compt. Rend. **157** 708-710, 1410-1413 (1913).

[2] A. Einstein, Ann. Physik **17**, 891 (1905).
English translation by W. Perrett and G.B. Jeffery in 'The Principle of Relativity' (Dover, New York, 1952) , p.37, or in 'Einstein's Miraculous Year' (Princeton University Press, Princeton, New Jersey, 1998) p.123.

[3] A.A. Michelson and E.W. Morley, Phil. Mag. **24**, 449 (1887).

[4] R.J. Kennedy, Proc. Nat. Acad. Sci. **12** 621 (1926).

[5] K.K. Illingworth, Phys. Rev. **30** 692 (1927).

[6] G. Joos, Ann. Physik **7**, 385 (1930).

[7] R.J. Kennnedy and E.M. Thorndike, Phys. Rev. **42** 400 (1932).

[8] D.C. Miller, Rev. Mod. Phys. **5** 203 (1933).

[9] R.S. Shankland *et al*, Rev. Mod. Phys. **27** 16 (1955).

[10] B. Pogany, Ann. Physik **80**, 217 (1926), **85**, 224 (1928).

[11] A.A. Michelson and H.G. Gale, Astrophys. J. **61**, 137, 140 (1925).

[12] W.M. Macek and D.T.M. Davis, Appl. Phys. Lett. **2**, 67 (1963).

[13] R.A. Bergh, H.C. Lefevre and H.J. Shaw, Optics Lett. **6**, 502 (1981).

[14] J.L. Davis and S. Ezekiel, Optics Lett. **6**, 505 (1981).

[15] ' Sensitive fiber-optic gyroscopes', Physics Today, Oct 1981 p.20.

[16] R. Wang *et al*, Physics Letters **A 312**, 7-10 (2003).

[17] R. Wang, Y. Zheng and A. Yao, Phys. Rev. Lett. **93** 143901 (2004).

[18] E.J. Post, Rev. Mod. Phys. **39** No 2 475-493 (1967).

[19] P. Langevin, Compt. Rendu. Acad. Sci. **173** 831 (1921).

[20] L. Silberstein, J. Opt. Soc. Am. **V** 291 (1921).

[21] A. Dufour and F. Prunier, Compt. Rendu. Acad. Sci.**204** 1925 (1937).

[22] F. Selleri, Foundations of Physics, **26** (5) 641 (1996)

[23] F. Selleri, Foundations of Physics Letters, **10** (1) 73 (1997)

[24] R.D. Klauber, Found. Phys. Lett. **16** (5) 447 (2003)

[25] A.Tartaglia in: 'Relativity in Rotating Frames', G.Rizzi and M.L. Ruggiero (Eds), (Kluwer Academuc Publishers, Dordecht) 2004, Ch 13 Section 4.

[26] P. Langevin, Scientia **10** 31 (1911) p41.

[27] J.H. Field, 'Langevin's 'Twin Paradox' paper revisited', arXiv pre-print: http://xxx.lanl.gov/abs/0811.3562. Cited 21 Nov 2008.

[28] N.D. Mermin, Am. J. Phys. **51** 1130 (1983).

[29] P. Langevin, Compt. Rendu. Acad. Sci. **200** 48 (1935).

[30] L.D. Landau and E.M. Lifshitz, 'The Classical Theory of Fields', 4th Ed (Pergamon Press, Oxford 1975) Chapter 10 §89 p254.

[31] P. Ehrenfest, Phys. Zeitschr.**10** 918 (1909).

[32] A. Einstein, Ann. der Physik, **49** 769 (1916)..

[33] A. Einstein, 'Relativity, the Special and the General Theory', English translation by R.W.Lawson, Methuen, London, 1960, Ch XXIII, P82, 'The Meaning of Relativity', Fifth Ed., Princeton University Press, Princeton, 1955, P59.

[34] G.Rizzi and M.L. Ruggiero (Eds) 'Relativity in Rotating Frames' (Kluwer Academuc Publishers, Dordecht) 2004

[35] P.Anderson, L.Vetharaniam and G.E. Stedman, Phys. Rep.. **295** 93 (1998).

[36] J.H. Field, Fundamental Journal of Modern Physics, Vol **2** Issue 2 139 (2011). arXiv pre-print: http://xxx.lanl.gov/abs/1210.2270. Cited 8 Oct 2012.

[37] L.B. Okun, K.G. Selivanov and V.L. Telegdi,[Usp. Fiz. Nauk **169** 1141 (1999)], Phys. Usp. **42** 1045 (1999).

[38] Y.T. Hay *et al* Phys. Rev. Lett. **4** 165 (1960).

[39] J.H. Field, 'The physics of space and time III: Classification of space-time experiments and the twin paradox', arXiv pre-print: http://xxx.lanl.gov/abs/0806.3671v2. Cited 7 Nov 2011.

[40] J.H. Field, 'Primary and reciprocal space-time experiments, relativistic reciprocity relations and Einstein's train-embankment thought experiment', arXiv pre-print: http://xxx.lanl.gov/abs/0807.0158v2. Cited 11 Nov 2011.

[41] J.H. Field,'Space-time attributes of physical objects and the laws of space-time physics', arXiv pre-print: http://xxx.lanl.gov/abs/abs/0809.4121 Cited 24 Sep 2008.

[42] J.C. Hafele and R.E. Keating, Science **177** 166-168, 168-170 (1972).

[43] R.P. Feynman, R.B. Leighton and M. Sands, 'Lectures on Physics' Volume I (Addison-Wesley, Reading, Massachusetts, 1964) Section 15-4.

[44] Ref. [43], Section 15-5.

[45] N. Ashby, 'Relativity and the global positioning system', Physics Today, May 2002, pp.41-47.

[46] D.W. Allan, M.A. Weiss and N. Ashby, Science **228** 64-70 (1985).

[47] P Wolf and G. Petit, Phys. Rev. **A56** 4405 (1997).

[48] K. Schwartzschild, Sitzungberichte Prüssiche Akademie der Wissenschaften p.198 (1916).

[49] S. Weinberg, 'Gravitation and Cosmology, Principles and Applications of the General Theory of Relativity', (John Wiley, New York, 1972) Ch 8 Section 2.

[50] I. Shapiro *et al*, Phys. Rev. Lett. **26** 1132 (1971).

[51] Y. Saburi, M.Yamamoto and K. Harada, IEEE Trans. Instr. Meas. **25** 473 (1976).

[52] C.-C. Su, Europhys. Lett. **56** (2) 170 (2001).

[53] C.-C. Su, Eur. Phys. J. Su **C 21** 701 (2001).

[54] Ed. K. Elsener, 'The CERN Neutrino beam to Gran Sasso (Conceptual Technical Design)', CERN 98-02, INFN/AE-98/05.

[55] OPERA Collaboration, R. Acquafrdda *et al* JINST **4** P04018 (2009).

[56] M.G. Kuhn, 'The influence of Earth rotation in neutrino speed measurements between CERN and OPERA detector', arXiv pre-print: http://xxx.lanl.gov/abs/1110.03920v2. Cited 20 Oct 2011.

[57] G. Colosimo *et al* 'Determination of the CNGS global geodesy', Opera public note 132 v2. Cited 10 Oct 2011; http://operaweb.lngs.infn.it/Opera/publicnotes/note132.pdf

[58] OPERA Collaboration, T. Adam *et al* 'Measurement of the neutrino velocity with the OPERA detector in the CNGS beam', arXiv pre-print: http://xxx.lanl.gov/abs/1109.4897v4. Cited 12 Jul 2012.

[59] J.H. Field, Eur. Phys. J. C **72** 2191 (2012). arXiv pre-print: http://xxx.lanl.gov/abs/1212.0001. Cited 30 Nov 2012.